

UNIVERSITY OF RUHUNA

Faculty of Engineering

End-Semester 4 Examination in Engineering: November/December 2016

Module Number: IS4301

Module Name: Probability and Statistics

[Three Hours]

[Answer all questions, each question carries fourteen marks]

Q1. a) A certain city divides naturally into ten subdivisions. How might a real estate appraiser select a sample of single family homes that could be used as a basis for developing an equation to predict appraised value from characteristics such as age, size, number of bathrooms, and distance to the nearest school?

[4.0 Marks]

b) If the appraiser has selected two samples for the characteristic "age" using different sampling methods as follows:

Sample A: 56, 32, 62, 66, 49, 53, 50, 65, 45, 56, 68, 38, 75, 57, 70

Sample B : 39, 42, 47, 40, 34, 35, 42, 36, 47, 49, 50, 37, 41, 48, 32

- Construct separate stem-and-leaf displays of the data samples.
- Find the sample mean, minimum value, maximum value and sample range of each sample.
- Describe similarities and differences for the two samples.

[6.0 Marks]

c) The system in Figure represents a configuration of solar photovoltaic arrays consisting of crystalline silicon solar cells. There are two sub systems connected in parallel, each one containing three cells. If components work independently of one another and the probability of component works is 0.9, calculate the probability of system works.

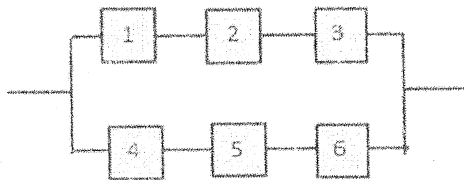


Figure: A system of solar cells connected in series - parallel

[4.0 Marks]

- Q2. a) Consider an experiment in which each of three vehicles taking a particular freeway exit turns left (L) or right (R) at the end of the exit ramp.
- What are the possible outcomes that comprise the sample space?
 - Give the possible sample points of the compounds events given bellow.
 A - the event that exactly one of the three vehicles turns right
 B - the event that at most one of the vehicles turns right
 C - the event that all three vehicles turn in the same direction
 - Assuming that the probability of randomly selected car taking a particular freeway exit turns left (L) is 0.25. If the random variable X denotes the number of vehicles taking exit turns left (L), then find the probability distribution of X .

[6.0 Marks]

- b) An appliance dealer sells three different models of upright freezers having 13.5, 15.9, and 19.1 cubic feet of storage space, respectively. Let X = the amount of storage space purchased by the next customer to buy a freezer. Suppose that X has probability mass function as given in Table.

Table: Probability mass function of X

X	13.5	15.9	19.1
$P(X = x)$	0.2	0.5	0.3

- Compute $E(X)$, $E(X^2)$, and $Var(X)$.
- If the price of a freezer having capacity X cubic feet is $25X - 8.5$, what is the expected price paid by the next customer to buy a freezer?
- What is the variance of the price $25X - 8.5$ paid by the next customer?
- Suppose that although the rated capacity of a freezer is X , the actual capacity is $h(X) = X - 0.01X^2$. What is the expected actual capacity of the freezer purchased by the next customer?

[4.0 Marks]

- c) Suppose that the random variable X has a Normal distribution with mean μ and standard deviation σ . Then the random variable Y has a linear function of X such that $Y = aX + b$.

- Find the distribution of the random variable Y .
- In an industrial process the diameter of a ball bearing is an important component part. The buyer sets specifications on the diameter to be 3.0 ± 0.01 cm. The implication is that no part falling outside these specifications will be accepted. It is known that in the process the diameter of a ball bearing has a normal distribution with mean 3.0 and standard deviation 0.005. On the average, how many manufactured ball bearing will be scrapped?

[4.0 Marks]

- Q3. a) Let X_1, X_2, \dots, X_n be a random sample from a distribution with mean μ and standard deviation σ .
- Find the distribution of sample mean.
 - If the random sample represents the length of life of light bulbs those collected from an electrical firm. Based on the past data, the light bulbs manufactured by the firm have a length of life that is approximately normally distributed, with mean equal to 800 hours and a standard deviation of 40 hours. Find the probability that a random sample of 16 bulbs will have an average life of less than 775 hours.

[5.0 Marks]

- b) Each of newly manufactured items is examined and the number of scratches per item is recorded, yielding the data given in Table.

Table: Number of scratches per item and frequencies

Number of scratches per item	0	1	2	3	4	5	6	7
Observed frequency	18	37	42	30	13	7	2	1

Let X = the number of scratches on a randomly chosen item, and assume that X has a Poisson distribution with parameter λ .

- Use Maximum Likelihood Method to find an estimator of λ .
- Show that the estimator of λ is an unbiased estimator.
- Compute the estimate of λ for the data.
- What is the standard deviation (standard error) of the estimator? Compute the estimated standard error.

[9.0 Marks]

- Q4. a) Specimens of soil were obtained from a site both before and after compaction. Tests on 10 pre-compaction specimens gave a mean porosity of 0.413 and a standard deviation of 0.0324. Tests on 20 post-compaction specimens gave a mean porosity of 0.340 and a standard deviation of 0.0469. These standard deviations are not significantly different. Porosity follows a Normal distribution.

- At the 5% level of significance, did the compaction correspond to a significant reduction in mean porosity?
- At the 5% level of significance, is the reduction in mean porosity significantly less than the desired reduction of 0.1?

[6.0 Marks]

- b) A sample of 300 cars having cellular phones and one of 400 cars without phones were tracked for 1 year. The Table gives the number of these cars involved in accidents over that year.

Table: Number of cars involved in accidents

	Accident	No Accident
Cellular phone	22	278
No phone	26	374

Use the above data to test the hypothesis that having a cellular phone in a car and being involved in an accident are independent. Use the 5% level of significance.

[8.0 Marks]

- Q5. Corrosion of steel reinforcing bars is the most important durability problem for reinforced concrete structures. Representative data on X = carbonation depth (mm) and Y = strength (MPa) for a sample of core specimens taken from a particular building are given in Table.

Table: Strength (Y) versus carbonation (X) depth for a sample of core specimens

x	8	15	16.5	20	20	27.5	30	30	35	38
y	22.8	27.2	23.7	17.1	21.5	18.6	16.1	23.4	13.4	19.5

- a) Construct a Scatter plot. Does a scatter plot support the choice of the simple linear regression model? Explain.
- b) Calculate the value of sample correlation coefficient and compare the result with part a).
- c) If the answer for part a) is "yes", then
- i determine the equation of the estimated regression line.
 - ii predict strength for carbonation depth value of 45.
 - iii calculate a 95% prediction interval for a strength value that would result from selecting a single core specimen whose carbonation depth is 45 mm (Hint: assume that the standard deviation of strength value is 2.68).

[9.0 Marks]