



UNIVERSITY OF RUHUNA

Faculty of Engineering

End-Semester 6 Examination in Engineering: November 2016

Module Number: IS6301

Module Name: Mathematical Modelling

[Three hours]

[Answer all questions, each question carries fourteen marks]

Q1.

a) Name two problems that might be modelled mathematically. Why do you think mathematics may provide a key to each solution? What is the added value in each case?

[2 Marks]

b) Scientists can determine the age of ancient objects by a method called radiocarbon dating. The bombardment of the upper atmosphere by cosmic rays converts nitrogen to a radioactive isotope of carbon, C-14, with a half life of about 5730 years. Vegetation absorbs carbon dioxide through the atmosphere and animal life assimilates C-14 through food chains. When a plant or animal dies it stops replacing its carbon and the amount of C-14 begins to decrease through radioactive decay. Therefore, the level of radioactivity must also exponentially decay. A parchment fragment was discovered that had about 74% as much C-14 radioactivity as does plant material on the earth today. Estimate the age of the parchment.

[6 Marks]

c) One dimensional simple harmonic motion for the system shown in Figure Q1 can be represented by a second order differential equation as below. The equations can be obtained using *Newton's second law of motion* ($F=ma$) and *Hooke's Law* ($F=-ku$).

$$m \frac{d^2 u_1}{dt^2} + 2ku_1 - ku_2 = 0$$

$$m \frac{d^2 u_2}{dt^2} - ku_1 + 2ku_2 = 0$$

Where; m = mass, u = displacement, k = spring constant and subscripts 1 and 2 denote particles 1 and 2 respectively.

Given that $k=1$ and $m=1$, find the natural frequencies of vibration of the system.

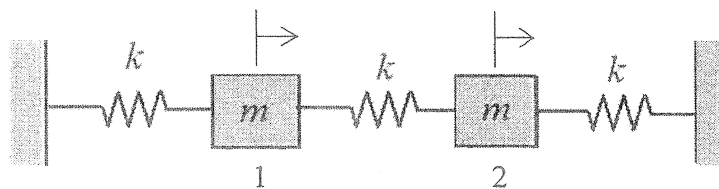


Figure Q1

[6 Marks]

Q2.

a) i.) Briefly explain the importance of 'Dimensional Analysis' in mathematical modelling of real world problems.

ii.) Clearly state the 'Buckingham Pi Theorem'.

[2 Marks]

b) The sound power (W), from a turbulent jet through a nozzle is believed to depend on the jet centerline velocity (V), nozzle diameter (D), speed of sound (C) and fluid density (ρ). The temperature and composition of the fluid in the jet is the same as the ambient fluid.

i.) Using the Buckingham Pi theorem, find a relation between the sound power and the other parameter(s) of the system.

ii.) How would the sound power vary if the nozzle diameter is doubled with all other factors remaining the same?

[4 Marks]

c) The advertising alternatives for a manufacturing company include television, radio and newspaper advertisements. The costs and estimates for audience coverage are given below.

| | <i>Television</i> | <i>Radio</i> | <i>Newspaper</i> |
|-------------------------------------|-------------------|--------------|------------------|
| <i>Cost per advertisement (Rs.)</i> | 200,000 | 60,000 | 30,000 |
| <i>Audience per advertisement</i> | 100,000 | 40,000 | 18,000 |

The local newspaper limits the number of weekly advertisements from a single company to ten. Moreover, in order to balance the advertising among the three types of media, no more than half of the total number of advertisements should occur on the radio, and at least 10% should occur on television. The weekly advertising budget is Rs. 1.82 million. Estimate the number of weekly advertisements that should be run in each of the three types of media to maximize the total audience, by using Simplex method.

[8.0 Marks]

Q3.

a) Briefly explain about 'Balanced and Unbalanced Transportation Problems' and the use of 'Dummy demand point and Dummy supply point' in transportation problems.

[2 Marks]

b) Briefly explain the procedures of following methods used to solve transportation problems.

i.) The Northwest Corner Method

ii.) The Stepping-Stone Solution Method

[2 Marks]

c) Oranges are grown, picked, and then stored in warehouses in Bibile, Badulla and Monaragala. These warehouses supply oranges to markets in Ampara, Ratnapura, Kandy and Galle. The following table shows the supply costs per truckload (Rs.1000s), supply and demand.

| From | To | | | | Supply |
|------------|--------|-----------|-------|-------|--------|
| | Ampara | Ratnapura | Kandy | Galle | |
| Bibile | 9 | 14 | 12 | 17 | 200 |
| Badulla | 11 | 10 | 6 | 10 | 200 |
| Monaragala | 12 | 8 | 15 | 7 | 100 |
| Demand | 130 | 170 | 100 | 150 | |

- i.) Set up the transportation tableau for this problem and determine the initial solution using the Northwest corner method.
- ii.) Find the total cost for the initial feasible solution.
- iii.) Solve the problem using the Stepping-Stone Solution Method.
- iv.) Are there multiple optimal solutions? If so, explain and identify them.

[8.0 Marks]

- d) Suppose, in part (c), the supplies are prohibited from Badulla to Kandy due to ongoing road construction works. What modification to the transportation tableau would you do to consider this prohibited root? Without repeating the calculations, briefly explain any specific points of solving this problem with this modification.

[2.0 Marks]

Q4.

- a) Briefly explain what is meant by,
 - i.) 'Row reductions' and 'Column reductions' performed in assignment problems.
 - ii.) 'Prohibited Assignment'.
- b) Give examples (2 each) for maximization and minimization assignment problems occurring in real world with related to engineering/management.
- c) A Company employs typists on hourly basis. There are five typists for service and their charges and speeds are shown below. According to an earlier understanding, only one job is given to one typist and is paid for a full hour even if the typist works for a fraction of an hour. The available five jobs are shown below. Find the least cost required to complete 5 jobs by assigning each typist a different job.

[2 Marks]

[2 Marks]

| Typist | Rate per hour (Rs.) | No. of pages Typed/ hour |
|--------|---------------------|--------------------------|
| A | 5 | 12 |
| B | 6 | 14 |
| C | 3 | 8 |
| D | 4 | 10 |
| E | 4 | 11 |

| Job | No. of pages |
|-----|--------------|
| P | 199 |
| Q | 175 |
| R | 145 |
| S | 198 |
| T | 178 |

[10 Marks]

Q5.

- a) Find the equilibrium point, if it exists, for the following dynamical system. Is the equilibrium point stable or unstable?

$$u(n) = 2u(n-1) + v(n-1) + 3$$

$$v(n) = 4u(n-1) - v(n-1) + 6$$

[2.0 Marks]

- b) Assume that,

- there are 2 chemicals in the body, U and V
- the body filters out 10% of U and 15% of V each day through the kidneys
- liver enzymes metabolize 30% of U into V and 25% of V into U each day.

In addition we assume that, each day x mg of V are taken in but no U is taken in.

- What should x be so that the equilibrium value for V is 200 mg?
- What will be the resulting equilibrium value for U given the prescribed dosage of V found in part (i)?

[6.0 Marks]

- c) Maria makes a single Rs. 50,000.00 purchase on her credit card and then never uses the card again. The card carries an 18% annual interest rate compounded monthly, and she pays back her minimum monthly payment of Rs. 1,000.00 per month.

- Write down a discrete dynamical system with initial condition to represent the amount Maria owes after n monthly payments.
- How many months will it take Maria to pay off this credit card debt?
- Explain what happens if Maria's single purchase was for Rs. 75,000.00 instead of Rs. 50,000.00.

[6.0 Marks]