UNIVERSITY OF RUHUNA

Faculty of Engineering

End-Semester 6 Examination in Engineering: November 2016

Module Number: ME 6302 **Module Name: Automatic Control Engineering**

[Three Hours]

[Answer **all questions**, each question carries **twelve** marks]

Important:
Some necessary equations and a partial table of Laplace transformation pairs have been provided on the question paper. You may make additional assumptions, if necessary, by clearly stating them in your answers. Some standard notations may have been used without defining them.

Q1. a) A height adjustable stabilized platform built with a shock-absorber system [$G(s)$] and a single gain [K] feedback path is shown in Figure Q1(i). In the system, the control input [$u(t)$] is adjusted dynamically in order to make the response [$y(t)$] to reach the reference height [$r(t)$]. The equivalent closed loop block diagram for the entire system is shown in the Figure Q1(ii). Parameters k and b stand for the spring and damper coefficients of the shock-absorber system, respectively.

i) Show that the open loop transfer function of the shock-absorber system is represented by, $G(s) = \dfrac{\eta}{s^2 + 2\sigma s + \rho}$
where $\sigma = b/2m$, $\rho = k/m$, $\eta = 1/m$

ii) Obtain the closed loop transfer function of the stabilized platform shown in Figure Q1 (ii).

iii) Find the poles of the characteristic equation and show that the platform achieves its marginal stability at $K = -\dfrac{\rho}{\eta}$.

iv) Show that the platform undergoes a critically damped stage at $K = \dfrac{\sigma^2 - \rho}{\eta}$.

v) Comment on the behavior of the system when; $-\dfrac{\rho}{\eta} < K < \dfrac{\sigma^2 - \rho}{\eta}$.

Given that the m=0.1 kg, b=1 Ns/m, k=2 N/m for the stabilized platform;

vi) Prove that the system shows an oscillatory stable response when the feedback gain [K] reaches to 4.

[8.0 Marks]

b) Calculate the natural un-damped frequency, peak overshoot and settling time for the platform described in part a-vi).

[2.0 Marks]

c) What would be the minimum value of the damper coefficient in order to maintain the settling time below 0.5 s?

[2.0 Marks]

Q1. is continued to Page 2.

Hint:
For under-damped generic second order systems;

The transfer function, $G(s) = \dfrac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$, peak overshoot; $PO = e^{\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}}$,

settling time; $T_s \approx \dfrac{4.6}{\zeta\omega_n}$ with usual notations.

Q2. a) *'Root Locus Design method can be used to locate poles at some desired locations. However, it is not possible to locate poles **arbitrarily**'.* Explain above statements giving reasons.
[2.0 Marks]

b) A unit step response of an open loop plant is shown in Figure Q2(b). Obtain the DC gain of the plant.
[1.0 Mark]

c) Following Matlab code is used to obtain the Root Locus [Figure Q2(c)] of the Plant.

```
close all;
clear all;
num=[1 6]; den=[1 3 2];
G=tf(num,den);
rlocus(G);
grid on;
```

Show the closed loop plant with a simple feedback gain(K) in a block diagram
[2.0 Marks]

d) With the help of Root Locus [Figure Q2(c)], determine
   i) Number of asymptotes and asymptote angle(s).
   ii) Asymptote intersection point(s).
   iii) Break away/in point(s).
[3.0 Marks]

e) Read from the Root Locus, the values of K for which the response is,
   i) critically damped and
   ii) most oscillatory damped
[2.0 Marks]

f) Comment on the value(s) of feedback gain (K) which the plant can show an unstable behavior?
[1.0 Mark]

g) What are the practical applications of critically damped controllers?
[1.0 Mark]

Q3. a) *'Single feedback gain tuning method is not always applicable in industrial plant control'.* Briefly explain the above statement and suggest a suitable modification.
[1.0 Mark]

b) A feedback loop of a space control system is shown in the Figure Q3.
   i) Obtain the Root Locus for the closed loop system.
   ii) Space control system demands 3% overshoot and 1s settling time. Show that the desired poles are located outside the system Root Locus.

Q3 is continued to Page 3.

iii) Design a lead compensator to move the Root Locus to the desired location and calculate the value of the feedback gain K in the desired system.

iv) Sketch the modified Root Locus and calculate new DC gain of the compensated closed loop system.

v) Introduce a front gain to adjust the response to unity DC gain and show the entire system in a block diagram.

[6.0 Marks]

c) Assume that there is an un-modeled pair of pure imaginary poles at $S = \pm j8$.

Design a suitable notch filter to eliminate oscillations generated due to pure imaginary poles. Re-adjust the DCG of the notch filtered system to unity and present the entire system in a block diagram.

[4.0 Marks]

d) Briefly explain this statement: *'In control system design, Lag compensators are less frequently used compared to Lead compensators'.*

[1.0 Mark]

Q4. A thermal power plant is modeled by a forward transfer function, $G(s) = \frac{1}{(S+1)^3}$.

To improve the plant's closed loop response, P-controller with a default gain of 3.16 is introduced and frequency response (Figure Q4) is analyzed. You are required to further improve the response by tuning P-controller and designing additional controller(s).

a) Modify Figure Q4 and show the frequency response of the plant without P-controller. State possible reason(s) [expectation(s)] for introducing a P-controller at the beginning?

[2.0 Marks]

b) What are the gain and phase margins of the P-controlled plant?

[1.0 Mark]

c) If P controller is tuned to achieve $\sqrt{3}$ *rad/s* bandwidth, what would be the tuned Kp Value?

[2.0 Marks]

d) Design a PD Controller to increase the bandwidth to $\sqrt{3}$ *rad/s*, overall phase margin to 30° and maximum steady-state error to 5%.

[3.0 Marks]

c) Show that PD controller is not able to satisfy all three criteria (bandwidth, phase margin and steady state error). Improve the plant's performance by introducing suitable modifications.

[2.0 Marks]

d) Draw the Matlab Simulink Diagram of the improved plant.

[2.0 Marks]

Hint: The solution to the equation $\phi_{\max} = \tan^{-1}\left(\frac{\omega_{\phi_{\max}}}{z_{le}}\right) - \tan^{-1}\left(\frac{z_{le}}{\omega_{\phi_{\max}}}\right)$ is given by

$$z_{le} = \frac{\omega_{\phi_{\max}}}{\tan\left(\frac{\phi_{\max} + 90^\circ}{z}\right)} \text{ with } \omega_{\phi_{\max}} = \sqrt{(z_{le}\, p_{le})}.$$

Q5. a) Figure Q5(a) shows a P-controller in a robot arm position control system. The P-controller generates the torque command, $u_p(t)=K_p e(t)$ for the motor which adjusts the arm position, $\theta(t)$. The weight of the arm creates a clockwise (-ve) torque, and this torque acts as a disturbance, $d(\theta(t))$. Assume K_p=25, M=3kg, L=1m, $g=9.8ms^{-2}$, and reference position, θ_r=1 *rad*.

i) Show that arm stabilizes at a steady state position, θ_{ss}=0.15 *rad*.

ii) Suggest a suitable controller to stabilize the arm with zero steady state error.

iii) Briefly discuss a major drawback of the proposed controller.

[4.0 Marks]

b) A plant shows an apparent dead time τ_d=0.6*s*, and apparent time constant τ=2*s*, and *DCGK*=0.7. It is required to design a PID Controller to achieve a stable response with an allowable phase margin and a gain margin. Standard PID tuning methods are used to obtain controller gains.

i) Calculate the PID gains using Cohen-Coon PID tuning method in Table Q5(i).

ii) Calculate ITAE PID gains using Integral-based PID tuning method in Table Q5(ii).

iii) Figure Q5(b) shows the unit step responses of the plant with and without controllers. Based on the calculated gain values above, identify responses [(1), (2), (3)] of the open loop plant, Cohen-Coon PID controlled plant and ITAE PID controlled plant.

[5.0 Marks]

c) Compare benefits and drawbacks of digital controllers over analog controllers?

[3.0 Marks]

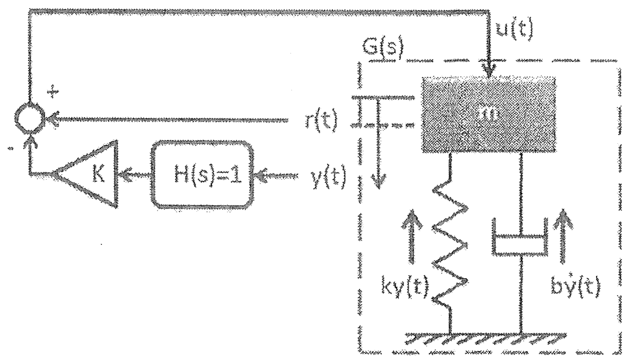

Figure Q1(i): Height Adjustable Stabilized Platform

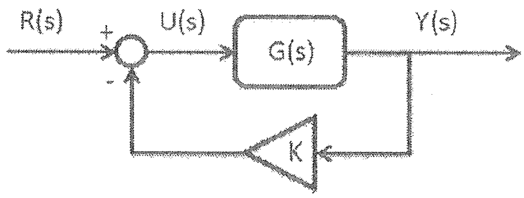

Figure Q1(ii): Equivalent Block Diagram of the Platform

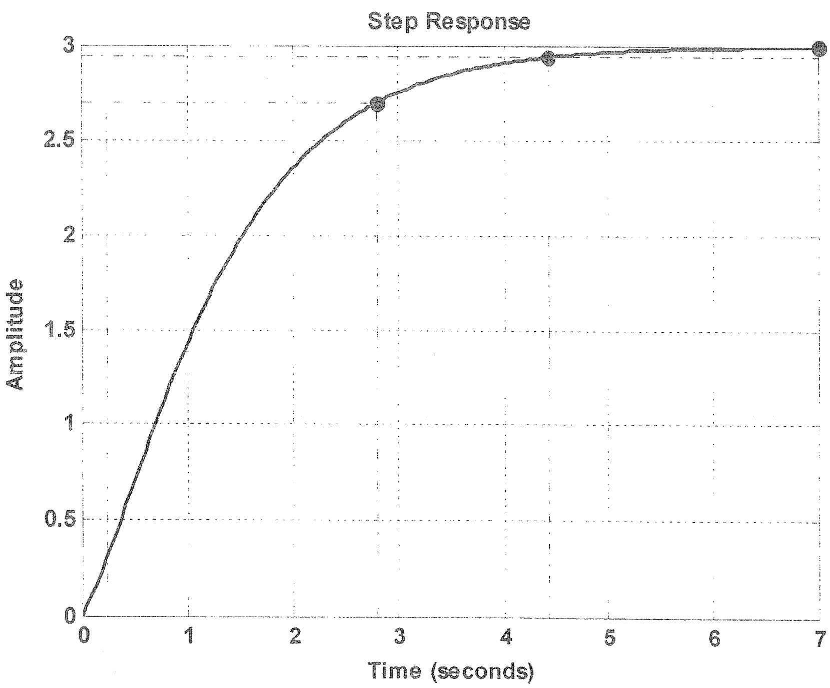

Figure Q2(b): Step Response of the Plant

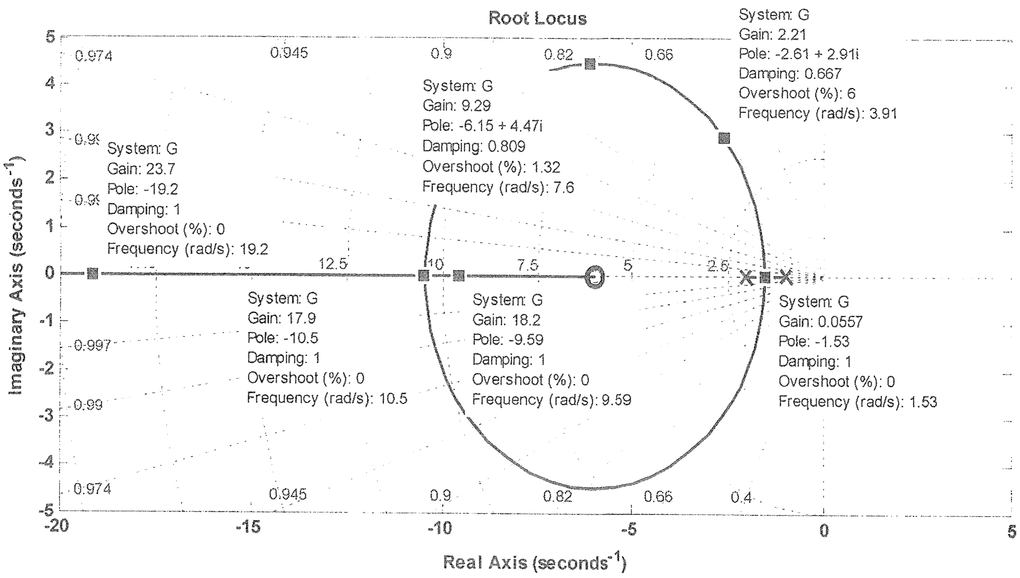

Figure Q2(c): Root Locus of the Plant

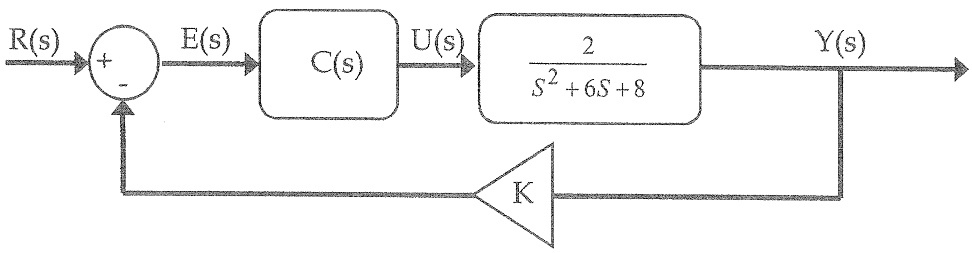

Figure Q3: Space Control System

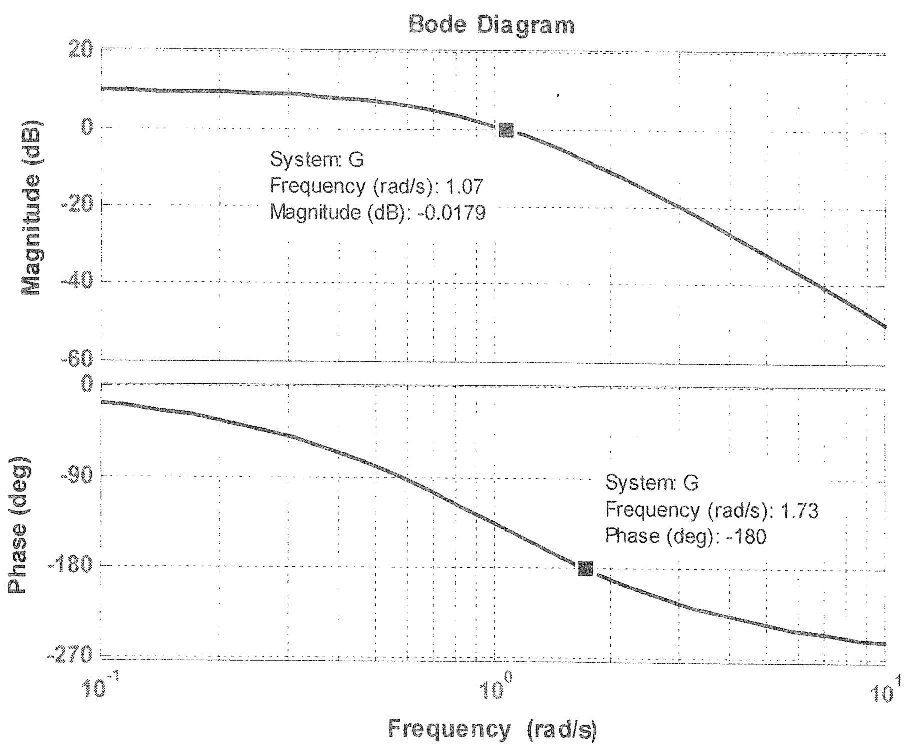

Figure Q4: Bode Diagram

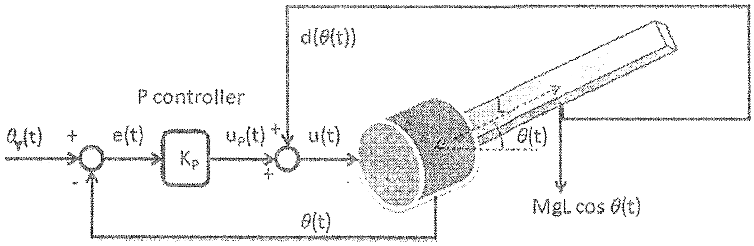

Figure Q5(a): P controller for Robot Arm Position Control

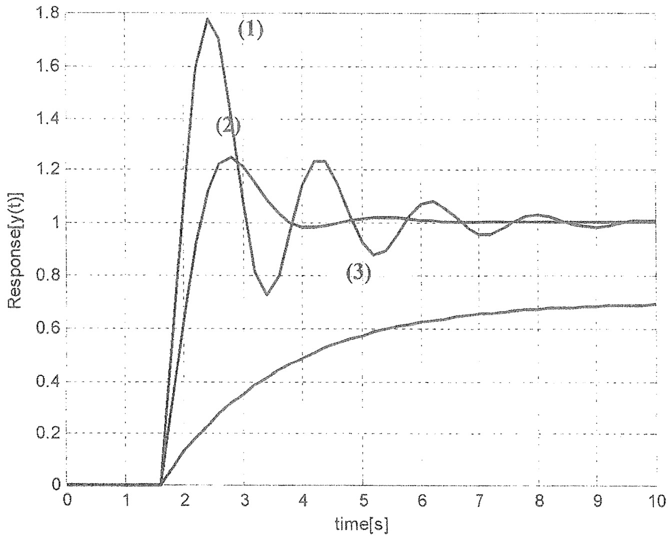

Figure Q5(b): Step Responses for Open Loop, Cohen-Coon PID Controlled and ITAE PID Controlled Plants

Table Q5(i): Cohen-Coon PID Gain Tuning Table

controller	K_P	T_I	T_D
P	$\frac{\tau}{DCG\tau_d}\{1+\frac{\tau_d}{3\tau}\}$	-	-
PI	$\frac{\tau}{DCG\tau_d}\{\frac{9}{10}+\frac{\tau_d}{12\tau}\}$	$\tau_d\{\frac{30+3\tau_d/\tau}{9+20\tau_d/\tau}\}$	-
PD	$\frac{\tau}{DCG\tau_d}\{\frac{5}{4}+\frac{\tau_d}{6\tau}\}$	-	$\tau_d\{\frac{6-2\tau_d/\tau}{22+3\tau_d/\tau}\}$
PID	$\frac{\tau}{DCG\tau_d}\{\frac{4}{3}+\frac{\tau_d}{4\tau}\}$	$\tau_d\{\frac{32+6\tau_d/\tau}{13+8\tau_d/\tau}\}$	$\tau_d\{\frac{4}{11+2\tau_d/\tau}\}$

Table Q5(ii): ITAE Based PID Gain Tuning Table

controller	Γ_D σ_1, σ_2	Γ_I σ_1, σ_2	Γ_D σ_1, σ_2
P	0.490,-1.084	-	-
PI	0.859,-0.977	0.674, -0.680	-
PID	1.357,-0.947	0.842, -0.738	0.381,0.995

Table of Laplace Transform Pairs

$f(t)$	$F(s)$
step	$\frac{1}{s}$
ramp,t	$\frac{1}{s^2}$
impulse	1
dirac delta function, $\delta(t-c);c\ge 0$	e^{-cs}
$u(t-a)$	$\frac{e^{-as}}{s}$
$u(t-a)\ g(t-a)$	$e^{-as}\,G(s)$
t^n	$\frac{n!}{s^{n+1}}$
e^{-at}	$\frac{1}{s+a}$
$t^n e^{-at}$	$\frac{n!}{(s+a)^{n+1}}$
$\frac{1}{(n-1)!}t^{n-1}e^{-at}$	$\frac{1}{(s+a)^n}$
$\sin\omega t$	$\frac{\omega}{s^2+\omega^2}$
$\cos\omega t$	$\frac{s}{s^2+\omega^2}$
$e^{-at}\sin\omega t$	$\frac{\omega}{(s+a)^2+\omega^2}$
$e^{-at}\cos\omega t$	$\frac{s+a}{(s+a)^2+\omega^2}$
$\frac{\omega_0}{\sqrt{1-\zeta^2}}e^{-\zeta\omega_0 t}\sin(\omega_0\sqrt{1-\zeta^2}t)$	$\frac{\omega_0^2}{s^2+2\zeta\omega_0 s+\omega_0^2}$
$1-\frac{\omega_0}{\sqrt{1-\zeta^2}}e^{-\zeta\omega_0 t}\sin(\omega_0\sqrt{1-\zeta^2}t+\phi),\quad Where,\ \phi=\tan^{-1}\left(\frac{\sqrt{1-\zeta^2}}{\zeta}\right)$	$\frac{\omega_0^2}{s(s^2+2\zeta\omega_0 s+\omega_0^2)}$
$\frac{d(f(t))}{dt}$	$sF(s)-f(0)$
$\frac{d^2(f(t))}{dt^2}$	$s^2F(s)-sf(0)-\dot{f}(0)$
$\int f(t)dt$	$\frac{1}{s}F(s)+\frac{1}{s}\left[\int f(t)dt\right]_{t=0}$
$f(t-\alpha)$	$e^{-\alpha s}F(s)\,with\,f(t-\alpha)=0,\ t\le$
$e^{-\alpha t}f(t)$	$F(s+\alpha)$
$f(t/a)$	$aF(as)$
Convolution Intergral, $\int_0^t f_1(t-\tau)f_2(\tau)d\tau$	$F_1(s)F_2(s)$