UNIVERSITY OF RUHUNA

BACHELOR OF SCIENCE HONOURS IN MARINE AND FRESH WATER SCIENCES DEGREE

Level I Semester I – August/September 2018

OCG 1141- Mathamatics I

Time: 2 hours

Answer ALL Questions. Calculators will be provided.

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1. a) Find the following limits:

   (i) $\lim_{t\to -3} \frac{2t+6}{4t^2-36}$,

   (ii) $\lim_{u\to 1} \frac{\sqrt{u}-1}{u-1}$.

   b) A population of butterflies in an enclosure at a zoo is modelled by

$$N(t) = 200 - \frac{140}{t+1},\ t \geq 0,$$

   where $N(t)$ is the number of butterflies t years after observations of the butterflies commenced.

   (i) How long will it take for the butterfly population to reach 172 butterflies?

   (ii) Find $\frac{dN(t)}{dt}$ using **first principles**.

   (iii) At what **rate** will the population be growing at the time when the butterfly population is 172?

   (iv) At what time will the growth rate be 10 butterflies per year?

   (v) Determine $\lim_{t\to\infty} N(t)$ and $\lim_{t\to\infty} \frac{dN(t)}{dt}$.

   c) Find the first derivative $f'(x)$ of $f(x) = \frac{\cos(3x)}{2e^x - x}$ and hence find the gradient at the point where $x = 0$.

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2. a) Determine the stationary points of the function

$$f(x) = 2 + 4x - 2x^2 - x^3$$

   and classify them as maxima or minima using the second derivative $f''(x)$.

   b) Show that the function

$$f(x, y) = x^2\, e^{x/y} + xy\, \cos(x/y)$$

   is homogeneous of degree 2 and satisfies the Euler's theorem.

c) Find A and B such that

$$\frac{7x+4}{6x^2+7x+2} = \frac{A}{2x+1} + \frac{B}{3x+2}.$$

Hence find

$$\int \frac{7x+4}{6x^2+7x+2}\,dx.$$

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3. a) Use **integration by parts formula** to show that

$$\int_0^{1/2} x\,e^{2x}\,dx = \frac{1}{4}.$$

b) The gradient of a curve is given by

$$\frac{dy}{dx} = ax - 6,$$

where a is a constant. Given the curve has a stationary point at $(-1, 10)$, determine its equation.

c) Apply the **method of separation of variables** to show that the general solution of the differential equation

$$\frac{dy}{dx} = y^2\,e^{2x},$$

given $y = 1$ when $x = 0$, is $y = \dfrac{2}{3 - e^{2x}}$.

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4. a) Show that the differential equation

$$y\,e^{xy}\,dx + (x\,e^{xy} + \sin y)\,dy = 0,$$

where $y = \pi$ when $x = 0$, is exact and find its solution.

b) Let

$$A = \begin{bmatrix} 3 & -1 \\ 5 & 4 \end{bmatrix}, \quad B = \begin{bmatrix} 2 & 6 \\ 1 & 7 \end{bmatrix}.$$

Using the above two matrices, show that matrix multiplication is not commutative in general.

c) Consider the system of linear equations

$$3x + 5y = 14$$
$$8x - 2y = 22.$$

(i) Write down the equivalent matrix equation $AX = B$ of the above system.

(ii) Find the inverse A^{-1} of A.

(iii) Use the above result in part (ii) to find x and y.
