UNIVERSITY OF RUHUNA

Faculty of Engineering

End-Semester 3 Examination in Engineering: August 2015

Module Number: ME 3305 **Module Name: Dynamics and Vibrations (Old Curriculum)**

[Three Hours]

[Answer **all questions**, each question carries **10** marks]

Q1. A packing crate is designed to protect a fragile instrument during shipment. Assuming that the packing material can be modeled as an ideal linear spring of stiffness k, in parallel with an ideal linear damper b, and that the instrument and crate are of mass, m_1 and m_2, respectively, the system can be modeled as shown in Figure Q1 (A).

The packing crate (with instrument inside) is dropped from a height h, as shown in Figure Q1 (B). The height is sufficiently large that by the time the crate hits the ground, the spring is fully extended to its unloaded length, Lo, as shown in Figure Q1(C). Note that the crate hits the ground with velocity, Vo , and in the presence of gravity.

a) Draw the free body diagram for the necessary parts of the system.

[2.0 Marks]

b) Derive the differential equation required for solving the system mathematically. Clearly indicate the initial conditions, and any inputs present.

[4.0 Marks]

c) For what values of b, does the instrument oscillate?

[4.0 Marks]

Q2. A system is defined by;

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

a) Determine the system eigenvalues and comment on the stability of the system.

[2.0 Marks]

b) Obtain an expression for the characteristic equation of the system.

[2.0 Marks]

c) Derive the transfer function matrix of the system.

[6.0 Marks]

Q3. a) Suppose a sinusoidal input ($u(t)=U_0 \sin \omega t$) has been applied on a dynamic system which the transfer function of the system is given by $G(s)$. If the system output is $y(t)$ with usual notations, Prove that:

$$y(t)=U_0 \left\| G(j\omega) \right\| \sin(\omega t+\varphi)$$

Briefly explain the significance of the above result.

[4.0 Marks]

b) A two wheel trailer of a tractor travels at a speed of 60 km/h over a road whose surface is sinusoidal with wave length of 20 m and amplitude of 30 mm as shown in Figure Q3. The mass of the trailer is 500 kg. It is supported by spring of total stiffness 25 N/mm and fitted with shock-absorber giving a damping ratio of 0.5. Considering only the single degree of freedom in the vertical direction, construct an equivalent mass spring dashpot with ground oscillation in order to simulate the trailer vibration. Hence, find the resulting amplitude of the trailer vibration.

[6.0 Marks]

Q4. The Figure Q4 shows a band pass filter.

a) Derive the differential equation which relates the input current I_s to the output voltage v_R. Show the steps in your derivation.

[2.0 Marks]

b) Use the result of above part (a) to derive the transfer function from input $I(s)$ to output $V_R(s)$.

[3.0 Marks]

c) In terms of the circuit parameters, find the values of the natural frequency ω_n and the damping ratio ζ. What are the locations of the system poles and zeros?

[3.0 Marks]

d) In the following parts of the problem, assume the components take the values: R = 50 kΩ, C = 10 pF, and L = 3.14815 x10^{-7} H. (Caution: The values chosen must be used with all given digits to guarantee a sufficiently accurate answer). For these parameter values, what are the numerical values of ω_n and ζ?

[2.0 Marks]

Q5. The governing equation of a simple pendulum of length L and total mass M with oscillated angle of θ is given by;

$$\ddot{\theta} + \frac{3C}{ML^2}\dot{\theta} + \frac{3g}{2L}\sin\theta = 0.$$

Where, C is the viscous frictional torque due to air resistance and bearing resistance, acts on the pendulum and g is the acceleration due to gravity.

a) Using state space representation, obtain the dynamic system model of the system.

[2.0 Marks]

b) Find all the equilibrium solutions of the system.

[4.0 Marks]

c) Investigate the stability of the equilibrium solutions.

[4.0 Marks]

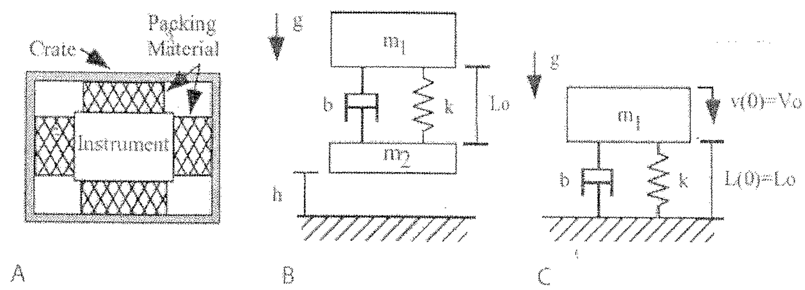

Figure Q1

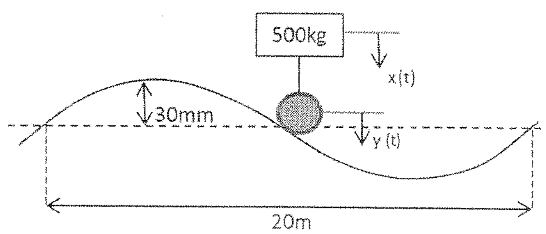

Figure Q3

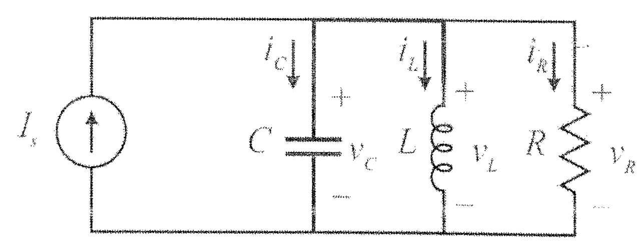

Figure Q4