

University of Ruhuna- Faculty of Technology
Bachelor of Engineering Technology, Information & Communication Technology and
Biosystems Technology
Level 1 (Semester 1) Examination, November 2019

Course Unit: TMS 1113 Foundation of Mathematics
Time Allowed 3 hours

Answer all Six (06) questions. Calculators are **not** allowed to use for calculations.

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1.1 Evaluate the followings.

- i. $\log_{10} 0.001$
- ii. $\ln e$
- iii. $\log_3 \frac{1}{81}$

1.2 i. Write $y = \log \frac{8 \times \sqrt[4]{5}}{81}$ in terms of $\log 2$, $\log 3$ and $\log 5$ to any base.

ii. If $\log 2 = 0.3010$, $\log 3 = 0.4771$ and $\log 5 = 0.6989$, find the value of y in above part (i).

1.3 Solve the equation $x^{3.2} = 41.15$, correct to 4 significant figures.

(Hints: $\log_{10} 41.15 = 1.6144$ and $10^{0.5045} = 3.195$)

1.4 The temperature θ_2 of a winding which is being heated electrically at time t is given by:

$\theta_2 = \theta_1(1 - e^{-t/\tau})$ where θ_1 is the temperature (in degrees Celsius) at time $t=0$ and τ is a constant. Calculate.

- i. θ_1 , correct to the nearest degree, when θ_2 is 50°C , t is 30 s and τ is 60 s.
- ii. Calculate the time t , correct to 1 decimal place, for θ_2 to be half the value of θ_1 , when τ is 60 s.

(Hints: $e^{-0.5} = 0.6065$ and $\ln 0.5 = -0.6931$)

1.5 Simplify the following expressions.

i. $(4 + 3i)(2 + 3i)$

ii. $\frac{(5 + 3i)}{(2 - i)}$

2 2.1 If $f(x) = \frac{x^2-16}{6}$ and $g(x) = \frac{24}{4x^2+20x+16}$ find the followings:

i. $f(x) + g(x)$,

ii. $f(x) - g(x)$,

iii. $f(x) \times g(x)$,

iv. $f(x)/g(x)$.

2.2 Solve the inequality.

$$\frac{t+1}{3t-6} > 0$$

2.3 Solve $2x^2 + 5x = 3$ by 'completing the square'.

2.4 Solve the following equations using the quadratic formula. Answer should be correct to 4 significant figures.

$$\frac{x+2}{4} + \frac{3}{x-1} = 7$$

2.5 A farmer has a field that is 400 m by 200 m. He is cutting the field in a spiral pattern, starting from the outside and working in towards the center. After an hour of work, 72% of the field is left uncut. What is the size of the ring cut around the outside?

3 3.1 There were 100 students in the library who responded to how they completed their research paper.

18 students only used the periodicals

29 students used the web and books

15 students used books, the web, and periodicals

40 students used books and periodicals

20 used the web and periodicals

60 students used books

7 students did not use the web, nor books, nor the periodicals

Find the followings.

i. Represent this information with a Venn diagram

ii. How many students used the web in their research?

iii. How many students used books or periodicals in their research?

iv. How many students used only books in their research?

v. How many students used web or periodicals in their research?

3.2 i. For any sets A, B, C prove $A \times (B \cap C) = (A \times B) \cap (A \times C)$

ii. Suppose $A = \{1, 2\}$, $B = \{p, q\}$ and $C = \{1, q\}$. Find

- a) $A \times B$
- b) $A \times C$
- c) $A \times (B \cap C)$

3.3 If $U = \{a, b, c, d\}$, $A = \{a, b, c\}$, $B = \{b, c, d\}$ and $C = \{a, d\}$ find the followings.

- i. $A \cap C'$
- ii. $B' \cap C$
- iii. $A' \cup C'$
- iv. $B' \cup C'$
- v. $A' \cap B'$
- vi. $A \cap B'$
- vii. $A' \cup B' \cap C'$
- viii. $B \cap A' \cup C'$

3.4 If $A = \{x : x \in \mathbb{N}, x < 5\}$,

- i. Write out the set A in roster notation
- ii. Find the cardinal number of a set A

4 4.1 Convert degrees to radians, in terms of π .

- i. 75°
- ii. 120°

4.2 Convert radians to degrees.

- i. $2\pi/5$
- ii. $7\pi/4$

4.3 Find all solutions in the given domain.

- i. $\cos x = \sqrt{3}/2$, $x \in [0, 2\pi]$

4.4 Rewrite each of the following as the *sin* or *cos* of a single angle and evaluate where possible.

- i. $\cos 40^\circ \cdot \cos 50^\circ - \sin 40^\circ \cdot \sin 50^\circ$
- ii. $\sin 110^\circ \cdot \cos 10^\circ - \cos 290^\circ \cdot \cos 80^\circ$

4.5 Prove the following identity.

$$\frac{\sin 2x + \cos 2x + 1}{\sin x + \cos x} = 2 \cos x$$

5 5.1 Let $A = \begin{bmatrix} -3 & 1 & 4 \\ 2 & -3 & 1 \\ 0 & 0 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0 & 3 \\ 1 & 1 & 1 \end{bmatrix}$, $C = \begin{bmatrix} 4 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$, $D = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$, $E = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$. Find the indicated following matrix if it is defined and explain your answer.

- i. $B - 4C$
- ii. ED
- iii. A^T
- iv. $B^T D$
- v. $A + B^T C$

5.2 Let $F = \begin{bmatrix} 2 & 5 \\ -3 & 1 \end{bmatrix}$ and $G = \begin{bmatrix} 8 & -10 \\ 6 & p \end{bmatrix}$. For what values of p , if any, will we have $FG = GF$?

5.3 Compute the inverse of following matrix R.

$$R = \begin{bmatrix} 1 & 4 & -3 \\ 2 & 7 & -6 \\ 1 & 8 & -2 \end{bmatrix}$$

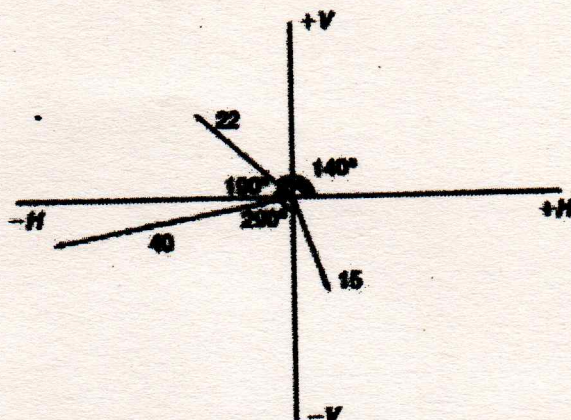
5.4 Using the above (Part 5.3) calculated inverse matrix (R^{-1}), solve the following sets of equations.

$$x + 4y - 3z = -4$$

$$2x + 7y - 6z = 1$$

$$x + 8y - 2z = -3$$

- 6 6.1 Velocities of 22 ms^{-1} , 40 ms^{-1} and 15 ms^{-1} act as shown in following figure. Calculate the magnitude of the resultant velocity and its direction relative to the horizontal.



(Hint: $\cos 40 = 0.7660$, $\cos 10 = 0.9848$, $\cos 70 = 0.3420$, $\sin 40 = 0.6428$, $\sin 10 = 0.1737$, $\sin 70 = 0.9396$)

- 6.2 For given vectors $\vec{p} = 2\mathbf{i} - \mathbf{j} + 2\mathbf{k}$, $\vec{q} = -\mathbf{i} + \mathbf{j} + \mathbf{k}$. Determine the unit vector in the direction of the vector $\vec{p} + \vec{q}$.
- 6.3 For vectors $\vec{a} = -3\mathbf{i} + 5\mathbf{j} + \frac{1}{2}\mathbf{k}$ and $\vec{b} = 2\mathbf{i} - 5\mathbf{j} - \frac{1}{3}\mathbf{k}$ find
- $\vec{a} \cdot \vec{b}$
 - $\vec{a} \times \vec{b}$
 - $|\vec{a} \times \vec{b}|$
 - $\vec{b} \times \vec{a}$
 - The angle between the vectors
- 6.4 Forces of $(2\mathbf{i} + \mathbf{j})$, $(-\mathbf{i} - 2\mathbf{j})$, $(3\mathbf{i} - 2\mathbf{j})$ newtons act at three points having position vectors of $(-\mathbf{i} + 5\mathbf{j})$, $(4\mathbf{i} + 5\mathbf{j})$ and $(-\mathbf{i} + \mathbf{j})$ meters, respectively. Calculate the magnitude of the moment. (Hint: Moment = Force \times distance)