# UNIVERSITY OF RUHUNA

## Faculty of Engineering

End-Semester 6 Examination in Engineering: January 2022

**Module Number: EE6302** **Module Name: Control System Design (C-18)**

**[Three Hours]**

[Answer **all questions**, each question carries 12 marks]

Note: Formulas you may require are given in page 5. A table of Laplace transforms is attached in page 6.

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Q1 a) i) Using necessary block diagrams, explain the terms, open-loop control and closed-loop control in control engineering.

ii) Describe the main advantage and the disadvantage of closed-loop control systems.

[3 Marks]

b) i) Drawing a suitable time response, explain the terms; rise time, settling time, maximum overshoot and peak time, associated with control systems.

ii) A unity feedback system is shown in Figure Q1(b). Assume that k>0. It is required to design the system so that it gets 5% maximum overshoot and 1 s peak time. Determine whether both specifications can be met simultaneously by selecting a value for k.

[4 Marks]

c) Consider the system shown in Figure Q1(c).

i) Clearly stating the conditions to be fulfilled, obtain the expression for the steady-state error of the system.

ii) If $G(s)=\frac{k}{s(4s+1)}$, where $k>1$, determine the value of $k$ so that system exhibits zero steady-state error to a unit step input. Calculate the time function of the unit step response of the system, if k = 4.

[5 Marks]

Q2 a) i) In terms of the characteristic equation of a system, what is the necessary condition to be fulfilled in order to have a stable system?

ii) State the Routh's necessary and sufficient condition to have a stable system.

[1.5 Marks]

b) Consider the system shown in Figure Q2(b). Use Routh's stability criterion to determine whether the system is stable or not. If the system is unstable, find how many roots have positive real parts.

[3.5 Marks]

c) i) Write the general form of matrix equations so that a system is represented in state-variable form. Name the matrices in your matrix equations.

ii) A system is described by the third-order differential equation $\dddot{y} + 6\ddot{y} + 11\dot{y} + 6y = 6u$, where $y$ and $u$ are the output and the input respectively. Taking the state vector $x$ as

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}, \text{ where }, x_1 = \ddot{y}, x_2 = \dot{y} \text{ and } x_3 = y,$$

represent the system in state-variable form.

iii) Consider the system described by

$$\begin{bmatrix} \dot{x_1} \\ \dot{x_2} \end{bmatrix} = \begin{bmatrix} -4 & -1 \\ 3 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u$$

$y = [1 \quad 0] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$, where all the notations have usual meaning. Obtain the transfer function of the system.

[7 Marks]

Q3 a) Consider the closed-loop system shown in Figure Q3.
Forward transfer function G(s) and feedback transfer function H(s) can be written in terms of their corresponding numerator and denominator factored polynomials as;

$G(s) = \frac{N_G(s)}{D_G(s)}$ and $H(s) = \frac{N_H(s)}{D_H(s)}$

K> 0 is a variable gain.

i) State the definition for the term "root locus".

ii) Obtain the closed-loop transfer function of the system in terms of $N_G(s)$, $D_G(s)$, $N_H(s)$ and $D_H(s)$.

iii) Hence, show that the root locus of the system begins at the poles of G(s)H(s) and ends at the zeros of G(s)H(s) as the gain K varies from zero to infinity.

[4.0 Marks]

b) Consider the closed loop control system shown in Figure Q3 with

$G(s) = \frac{1}{(s+1)(s^2+14s+40)}$ and $H(s) = 1$

The input R(s) is a unit step. K> 0 is the variable gain.

i) Create the root locus of this system.

ii) Calculate the imaginary axis crossings of the root locus and mark the values appropriately on the root locus.

iii) Find the range of gain K, where the closed-loop system is stable. State reasons for your answer.

iv) Design the value of gain K to yield a 2% overshoot in the response of the closed-loop system assuming a second order underdamped system with two poles. Include the following in your solution.

- Required damping ratio $\xi$. Mark the necessary constant damping ratio line on the root locus and the angle it makes with the imaginary axis.

- A justification for the validity of above assumption in your design.

[8.0 Marks]

Q4 a) Consider the closed-loop control system shown in Figure Q4 (a) where

$$G(s) = \frac{K}{(s+p_1)(s+p_2)}$$

$p_1, p_2$ can be real or complex numbers. The gain K>0 is a real number.
$G_c(s)$ is the compensator transfer function. The input R(s) is a unit step.

i) Obtain expressions for the complex poles of the closed-loop system in terms of $p_1$, $p_2$ and K for the following cases;
   - Case A: considering an uncompensated system, i.e. $G_c(s) = 1$
   - Case B: considering that the compensator is purely a derivative compensator where $G_c(s) = s$

ii) Based on your answers in part a) i), comment on the effect of derivative compensation on settling time and peak time of the closed-loop system response.
Use a diagram, indicating the relative positions of the poles under two cases, to support your explanation.

[4.0 Marks]

b) **Answer this question using your knowledge on root locus design technique.** However, it is **NOT** necessary to create an accurate plot of the root locus of the given system.
Consider the closed-loop system shown in Figure Q4(b) where

$$G(s) = \frac{K}{s^3 + 52s^2 + 100s}$$

The input R(s) is a unit step.
The gain K has been adjusted to K = 177.62 to achieve a 15% overshoot in the closed loop system response, assuming a second order underdamped system with two poles.

i) Evaluate the validity of the second order approximation that has been used to calculate the gain K.

ii) Estimate the settling time and the peak time of the system response.

iii) Design a lead compensator to improve the settling time of the system response by a factor of 5 without significantly affecting the percent overshoot specification (15%).
Choose the zero of the compensator at s = -5

[8.0 Marks]

Q5 a) i) Create bode plots for the transfer function $G_{OL}(s)$ given below in the form of asymptotic approximations. Consider that K = 20.

$$G_{OL}(s) = \frac{K\,s}{(s+1)(s+20)}$$

ii) Assume that K is increased to 100.
State what movement can be expected in the magnitude **and** phase frequency responses you have developed. (left/ right/ up/ down/ no movement)

[4.0 Marks]

b) **Answer this question using your knowledge on frequency response design technique.** However, it is **NOT** necessary to create an accurate plot of the frequency response of the given system.

Consider the closed-loop system shown in Figure Q5, where

$$G(s) = \frac{K}{(s+5)(s^2+21s+20)}$$

The input R(s) is a unit step.

The gain K has been adjusted to K = 479.73 so that the closed loop system response operates with a 10% overshoot.

However, the steady state error of the system output was found to be too high in this design.

i) Calculate the steady state error of the system response.

ii) Determine a new value for the gain K to improve the steady state error by a factor of 10.

iii) Evaluate the stability of the closed-loop system at the new value for the gain K, calculated in part b) ii).

iv) Design a lag compensator so that the steady state error is improved by a factor of 10 without significantly affecting the percent overshoot specification (10%).

[8.0 Marks]

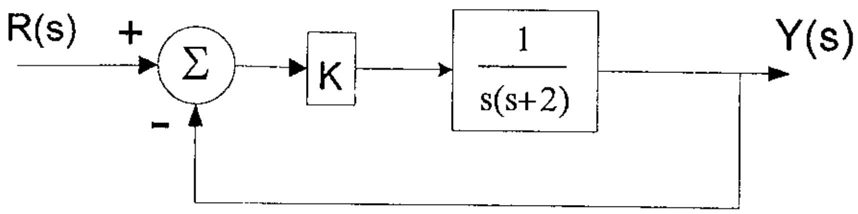



Figure Q1(b)

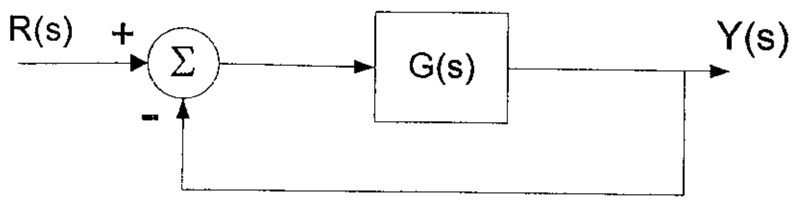



Figure Q1(c)

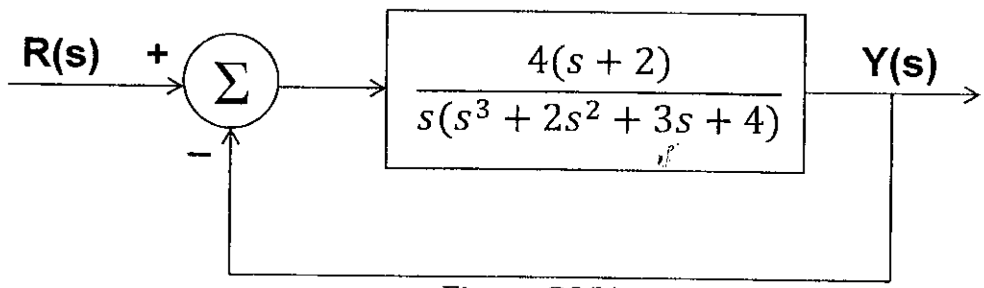



Figure Q2(b)

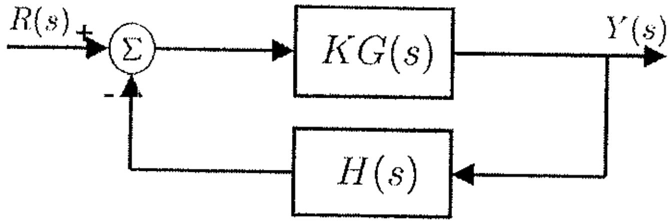



Figure Q3

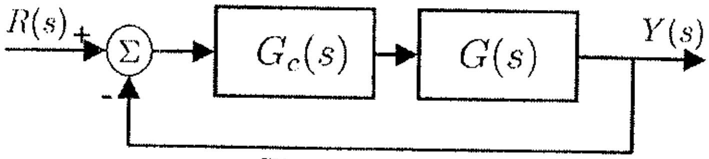



Figure Q4 (a)

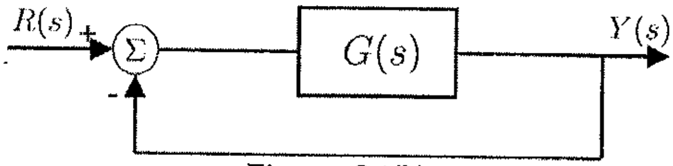



Figure Q4(b)

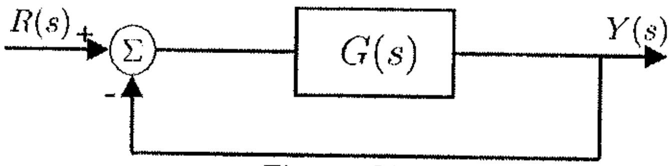
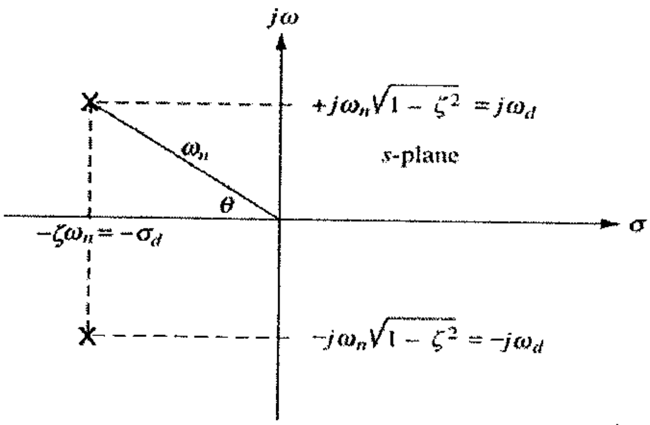



Figure Q5

Formulas you may require:
(All notations have their usual meaning)




$$T_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}} = \frac{\pi}{\omega_d}$$

$$T_s = \frac{4}{\zeta\omega_n} = \frac{4}{\sigma_d}$$

$$\%OS = e^{-\left(\frac{\zeta\pi}{\sqrt{1-\zeta^2}}\right)} \times 100$$

$$\phi_M = tan^{-1}\left(\frac{2\xi}{\sqrt{-2\xi^2+\sqrt{1+4\xi^4}}}\right)$$

$$\omega_{BW} = \frac{4}{T_s\xi}\sqrt{(1-2\xi^2)+\sqrt{4\xi^4-4\xi^2+2}}$$

**Table of Laplace Transforms**

| Number | $F(s)$ | $f(t), t \geq 0$ |
|---|---|---|
| 1 | $1$ | $\delta(t)$ |
| 2 | $\frac{1}{s}$ | $1(t)$ |
| 3 | $\frac{1}{s^2}$ | $t$ |
| 4 | $\frac{2!}{s^3}$ | $t^2$ |
| 5 | $\frac{3!}{s^4}$ | $t^3$ |
| 6 | $\frac{m!}{s^{m+1}}$ | $t^m$ |
| 7 | $\frac{1}{(s+a)}$ | $e^{-at}$ |
| 8 | $\frac{1}{(s+a)^2}$ | $te^{-at}$ |
| 9 | $\frac{1}{(s+a)^3}$ | $\frac{1}{2!}t^2e^{-at}$ |
| 10 | $\frac{1}{(s+a)^m}$ | $\frac{1}{(m-1)!}t^{m-1}e^{-at}$ |
| 11 | $\frac{a}{s(s+a)}$ | $1-e^{-at}$ |
| 12 | $\frac{a}{s^2(s+a)}$ | $\frac{1}{a}(at-1+e^{-at})$ |
| 13 | $\frac{b-a}{(s+a)(s+b)}$ | $e^{-at}-e^{-bt}$ |
| 14 | $\frac{s}{(s+a)^2}$ | $(1-at)e^{-at}$ |
| 15 | $\frac{a^2}{s(s+a)^2}$ | $1-e^{-at}(1+at)$ |
| 16 | $\frac{(b-a)s}{(s+a)(s+b)}$ | $be^{-bt}-ae^{-at}$ |
| 17 | $\frac{a}{(s^2+a^2)}$ | $\sin at$ |
| 18 | $\frac{s}{(s^2+a^2)}$ | $\cos at$ |
| 19 | $\frac{s+a}{(s+a)^2+b^2}$ | $e^{-at}\cos bt$ |
| 20 | $\frac{b}{(s+a)^2+b^2}$ | $e^{-at}\sin bt$ |
| 21 | $\frac{a^2+b^2}{s[(s+a)^2+b^2]}$ | $1-e^{-at}\left(\cos bt+\frac{a}{b}\sin bt\right)$ |