



# UNIVERSITY OF RUHUNA

Faculty of Engineering

End-Semester 4 Examination in Engineering: December 2015

Module Number: EE4301

Module Name: Communication Theory

[Three Hours]

[Answer all questions, each question carries 10 marks]

All the notations have their usual meanings.

- Q1 a) i) Illustrate the components in a typical communication system and state their functions.  
ii) What is the requirement for modulation in communication systems?  
[2.0 Marks]
- b) i) Derive the frequency domain expression for a standard Amplitude Modulated (AM) signal under tone modulation.  
ii) Sketch the spectrum of the AM signal.  
[1.5 Marks]
- c) i) Modulation based on switching processes is one method of producing a modulated signal. State two such amplitude modulators.  
ii) Describe the functionality of one such modulator mentioned in part i).  
[2.5 Marks]
- d) Consider a non-linear amplitude modulator. If the output characteristic of the nonlinear element is given by
- $$v_{out} = a v_{in} + b v_{in}^2$$
- i) Derive the expressions for  $v_{out}(t)$  and  $V_{out}(f)$  when
- $$m(t) = 3 \cos(500 \pi t)$$
- $$c(t) = \cos(5000 \pi t)$$
- Hint: Neglect the negative frequency components.
- ii) Determine  $a$  and  $b$ , if the amplitudes of spectral components at 2.5 kHz and 5 kHz are 3 and 4 respectively.
- iii) Sketch the frequency spectrum for  $v_{out}(t)$ .  
[4.0 Marks]

Q2 a) The general form of a Single Sideband (SSB) modulated signal can be given as

$$m_{c(SSB)}(t) = m(t)\cos\omega_c t \pm m_h(t)\sin\omega_c t$$

where  $m_h(t)$  is the Hilbert Transform of  $m(t)$ .

- i) State the methods that are used to generate SSB modulated signals and describe one of them briefly.
- ii) What is the significance of Hilbert transform in SSB systems?
- iii) Derive the expressions for Upper Sideband (USB) and Lower Sideband (LSB) for tone modulation condition.

[4.5 Marks]

- b) i) How does the Quadrature Amplitude Modulation (QAM) scheme utilize the bandwidth of the frequency spectrum?
- ii) Suggest a scheme to recover the QAM signal

$$m_{c(QAM)}(t) = m_1(t)\cos\omega_c t + m_2(t)\sin\omega_c t$$

[2.5 Marks]

- c) Consider the following message signals.

$$m_1(t) = 5\cos(100\pi t) + 10\cos(500\pi t)$$

$$m_2(t) = 10\sin(500\pi t)$$

If these messages are modulated using QAM with a carrier of frequency 2 kHz

- i) determine the modulated signal.
- ii) sketch the spectrum for the modulated signal.
- iii) compute the transmission power of the modulated signal.

[3.0 Marks]

- Q3 a) i) What is the difference between narrowband and wideband Frequency Modulation (FM)?
- ii) Describe the direct approach of generating FM waves.
- iii) State the different demodulation techniques that are used for FM.

[2.5 Marks]

- b) i) Explain the functionality of Pre-emphasis and De-emphasis filters.
- ii) The received signal at a receiver, the un-modulated carrier and the interference signal are denoted by  $R(t)$ ,  $C(t)$  and  $I(t)$  respectively. Show that the envelop of the received signal is

$$A_R(t) = (A_C + A_I)^2 + 2A_C A_I (\cos\omega_I t - 1)$$

where,  $C(t) = A_C \cos\omega_C t$

$$I(t) = A_I \cos(\omega_C + \omega_I)t$$

$$R(t) = A_R(t) \cos[\omega_C t + \phi_R(t)] = C(t) + I(t)$$

[2.5 Marks]

c) A FM modulator with characteristics  $A_c = 3$ ,  $f_c = 4$  kHz and  $k_f = 80\pi$  is connected to an input  $m(t) = 5 \cos(80\pi t)$ .

- i) Determine the modulation index  $\beta$ .
- ii) Sketch the frequency spectrum of the modulator output. Use the following third order Bessel function in Table Q3 to obtain this result.

$$m_{FM}(t) = A_c \sum_{n=-3}^3 J_n(\beta) \cos(\omega_c + n\omega_m)t$$

Hint:  $J_{-n}(\beta) = (-1)^n J_n(\beta)$

- iii) Compute the power of the modulated signal.

[5.0 Marks]

Q4 a) In Ideal sampling, an analog signal  $x(t)$  is multiplied with a periodic impulse train  $S_s(t)$ .

- i) Derive the frequency domain expression for the sampled signal  $X_s(f)$ .

Hint: The natural sampled signal is

$$X_s(f) = \sum_{n=-\infty}^{\infty} f_s \tau \operatorname{sinc}(n f_s \tau) X(f - n f_s)$$

where  $\tau$  is the duration of the rectangular pulse in natural sampling.

- ii) Consider the Ideal Sampled signal with  $f_s = 70$  Hz and  $x(t) = 1 + 2 \cos(2\pi 30t)$ . Obtain an expression for the sampled output in frequency domain.

[3.5 Marks]

b) A signal  $x(t)$  is band limited to  $f_m$  Hz. It can be reconstructed completely from its samples by passing the sampled signal through an ideal low pass filter. The cutoff frequency of the low pass filter is greater than  $f_m$  Hz.

- i) Derive an expression for the Interpolation filter output in time domain. Assume that the interpolation filter is an ideal low pass filter with a Gain  $k$  and a cutoff frequency  $B$  Hz.
- ii) Explain the method of reconstructing the message signal using the derived expression in part i).

[3.5 Marks]

c) Sketch the transmitted signal corresponding to the bit stream 1, 0, 1, 1, 1, 0, 0, 1 for each of the following line coding scheme. Assume that the channel is a low-pass linear time invariant system with a larger bandwidth.

- i) Bipolar Non Return to Zero (NRZ)
- ii) Polar Return to Zero (RZ)
- iii) Unipolar NRZ
- iv) Split-phase Manchester

[3.0 Marks]

- Q5 a) AT & T multiplexing hierarchy used in a digital telecommunication has four levels T1, T2, T3 and T4 lines. The signals of T1 line consists of Pulse Code Modulated (PCM) voice and multiplexed digital data.
- i) Draw the frame structure of T1 line and briefly explain it.
  - ii) If the PCM / Time Division Multiplexing (TDM) system uses a sampling frequency of 8 kHz, determine bit rate of T1 line.
  - iii) Explain why the output bit rate at a given level exceeds the sum of the input bit rate in the AT & T hierarchy.

[4.0 Marks]

- b) Consider an analog signal with values ranging from (-1.5) to (1.5) volts bandwidth of the signal is 6 kHz. This signal is sampled, quantized and binary coded to generate the PCM signal.
- i) Determine the step size of the quantized signal, if the samples are quantized into 16 uniform levels.
  - ii) Determine the number of digits in the codeword and design binary codewords for each quantization level.
  - iii) Determine the binary pulse rate (in bits per second) of the binary coded signal and the minimum bandwidth that requires to transmit the signal.
  - iv) If the signal is transmitted as a 4-ary PSK carrier modulated PCM signal, what is the minimum number of 4-ary pulses required to encode each sample?
  - v) What is the bandwidth of the signal after 4-ary PSK carrier modulation?

[6.0 Marks]

Table Q3

$x$ ( $m_f$ )	(CARRIER) $J_0$	H OR ORDER (-----Sidebands-----)															
		$J_1$	$J_2$	$J_3$	$J_4$	$J_5$	$J_6$	$J_7$	$J_8$	$J_9$	$J_{10}$	$J_{11}$	$J_{12}$	$J_{13}$	$J_{14}$	$J_{15}$	$J_{16}$
0.00	1.00	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
0.25	0.98	0.12	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
0.5	0.94	0.24	0.03	—	—	—	—	—	—	—	—	—	—	—	—	—	—
1.0	0.77	0.44	0.11	0.02	—	—	—	—	—	—	—	—	—	—	—	—	—
1.5	0.51	0.56	0.23	0.06	0.01	—	—	—	—	—	—	—	—	—	—	—	—
2.0	0.22	0.58	0.35	0.13	0.03	—	—	—	—	—	—	—	—	—	—	—	—
2.5	-0.05	0.50	0.45	0.22	0.07	0.02	—	—	—	—	—	—	—	—	—	—	—
3.0	-0.26	0.34	0.49	0.31	0.13	0.04	0.01	—	—	—	—	—	—	—	—	—	—
4.0	-0.40	-0.07	0.36	0.43	0.28	0.13	0.05	0.02	—	—	—	—	—	—	—	—	—
5.0	-0.18	-0.33	0.05	0.36	0.39	0.26	0.13	0.05	0.02	—	—	—	—	—	—	—	—
6.0	0.15	-0.28	-0.24	0.11	0.36	0.36	0.25	0.13	0.06	0.02	—	—	—	—	—	—	—
7.0	0.30	0.00	-0.30	-0.17	0.16	0.35	0.34	0.23	0.13	0.06	0.02	—	—	—	—	—	—
8.0	0.17	0.23	-0.11	-0.29	-0.10	0.19	0.34	0.32	0.22	0.13	0.06	0.03	—	—	—	—	—
9.0	-0.09	0.24	0.14	-0.18	-0.27	-0.06	0.20	0.33	0.30	0.21	0.12	0.06	0.03	0.01	—	—	—
10.0	-0.25	0.04	0.25	0.06	-0.22	-0.23	-0.01	0.22	0.31	0.29	0.20	0.12	0.06	0.03	0.01	—	—
12.0	0.05	-0.22	-0.08	0.20	0.18	-0.07	-0.24	-0.17	0.05	0.23	0.30	0.27	0.20	0.12	0.07	0.03	0.01
15.0	-0.01	0.21	0.04	-0.19	-0.12	0.13	0.21	0.03	-0.17	-0.22	-0.09	0.10	0.24	0.28	0.25	0.18	0.12

Source: E. Cambi, *Bessel Functions*, Dover Publications, Inc., New York, 1948. Courtesy of the publisher.