The genus of maximal embedding of the generalized Petersen graph, $GP\ (n,k)$ for the cases $k\ =\ 1,2$

Caldera P.A.D.S.P.[1]*, Almeida S.V.A.[1] and Wijesiri G.S.[1]

[1]*Department of Mathematics, University of Kelaniya, Dalugama, Kelaniya, Sri Lanka.*

In Topological graph theory, the maximal genus of graphs has been a fascinating subject. For a simple connected graph G, the maximal genus $\gamma_M\ (G)$ is the largest genus of an orientable surface on which G has a 2-cell embedding. $\gamma_M\ (G)$ has the upper bound $\gamma_M\ (G) \leq \left\lfloor \frac{\beta(G)}{2} \right\rfloor$, where $\beta(G)$ denotes the Betti number and G is said to be upper embeddable if the equality holds. In this study, the maximal genus of $GP(n,k)$ is established as $\gamma_M\ (GP\ (n,k)) = \left\lfloor \frac{n+1}{2} \right\rfloor$ for $k\ =\ 1$ and $k\ =\ 2$ by proving the upper embeddability of generalized Petersen graph, $GP\ (n,k)$ for the cases $k\ =\ 1$ and $k\ =\ 2$. The proof is done by obtaining spanning trees T and examining the components in the edge complements $GP(n,k)\backslash T$ for the cases $k\ =\ 1$ and $k\ =\ 2$ of $GP\ (n,k)$.

Keywords: Generalized Petersen graph, Maximum genus, Upper embeddability, Spanning tree, Splitting tree

*Corresponding author: sandunicaldera97@gmail.com