UNIVERSITY OF RUHUNA

Faculty of Engineering

End-Semester 5 Examination in Engineering: August 2015

Module Number: ME5301 **Module Name: Computer Aided Design**

[Three Hours]

[Answer **all** questions. All questions carry equal marks]

Q1. a) What are homogeneous coordinates?

[2.0 Marks]

b) Derive the transformation matrix for rotation of a point in the xy plane around z-axis.

[3.0 Marks]

c) I. Consider a triangle whose vertices are (2, 2), (4, 2) and (4, 4). Find the concatenated transformation matrix and the transformed vertices for rotation of 90^0 about the origin followed by reflection through the line $y = -x$.

II. Rotate the rectangle (shown in Figure Q1) formed by points A (1, 1), B (2, 1), C (2, 3), and D (1, 3) 30^0 counter clock wise about the point (3, 2) and find the new coordinates of the rectangle after rotation.

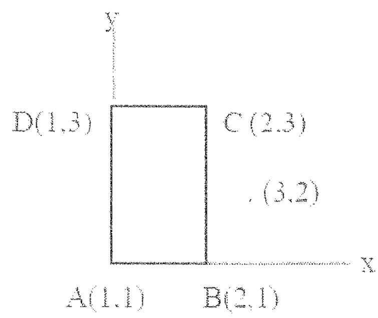

Figure Q1

[7.0 Marks]

Q2. a) I. Express the PC curves in terms of geometric boundary conditions.

II. For points A (1,2) , B (3,1) with corresponding slopes 60^0 and 30^0, show that the formulation of the curve is given by,

$$x(u) = 1 + 0.5u + 4.13u^2 - 2.63u^3,$$

$$y(u) = 2 + 0.86u - 5.23u^2 + 3.36u^3$$

[7.0 Marks]

b) A PC curve was derived in class for end point positions and tangent constraints, but these aren't the only geometric constraints that could be used. Develop a similar PC curve for end points and curvature constraints. (i.e. develop matrix equations)

$$\text{Take: } \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 2 & 0 & 0 \\ 6 & 2 & 0 & 0 \end{bmatrix}^{-1} = \begin{bmatrix} 0 & 0 & -1/6 & 1/6 \\ 0 & 0 & 1/2 & 0 \\ -1 & 1 & -1/3 & -1/6 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

[5.0 Marks]

Q3. a) Show that the element stiffness matrix (in global co-ordinates) of a spring element located at an angle θ to the horizontal as $k\begin{bmatrix} C^2 & CS & -C^2 & -CS \\ CS & S^2 & -CS & -S^2 \\ -C^2 & -CS & C^2 & CS \\ -CS & -S^2 & CS & S^2 \end{bmatrix}$.

[4.0 Marks]

b) The truss shown in Figure Q3 is subjected to a horizontal force F at node 4. Let all the members have the same stiffness EA. Find the displacement (in terms of F, E, A and L) at node 4.

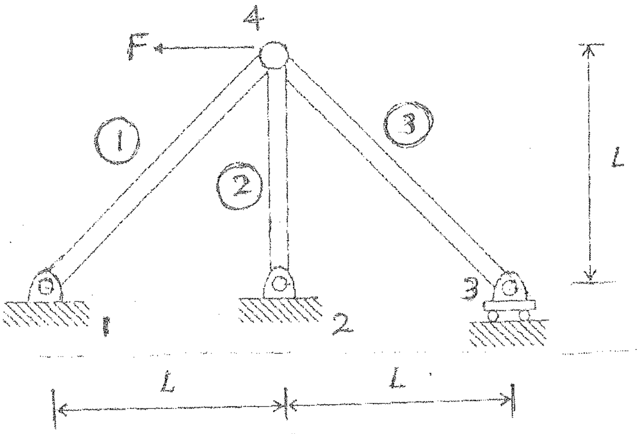

Figure Q3

[8.0 Marks]

Q4. a) Show that the stiffness matrix of triangular element (CST element) is given by $k = B^T DBtA$ with usual notation.

[4.0 Marks]

b) Derive the stiffness matrix and system of equations for the triangular plate shown in Figure Q4 using a CST element. E=200 GPa, υ=0.3, thickness=5 mm, P_1=5 N/mm^2 acting on side jk on x direction, P_2=2 N/mm^2 acting on side ik and perpendicular to side ik, Consider plane stress conditions.

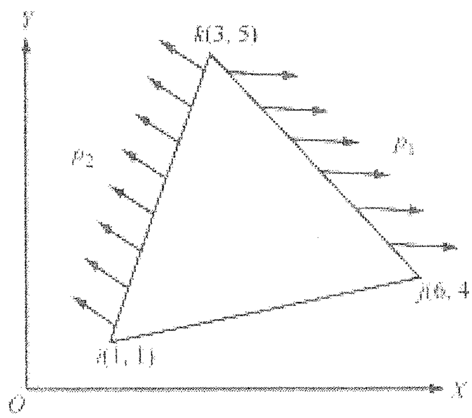

Figure Q4 The coordinates in mm $i(1,1)$, $j(6,4)$ and $k(3,5)$

The following are given.

- Constant values $a_1 = x_2y_3 - x_3y_2$, $a_2 = x_3y_1 - x_1y_3$, $a_3 = x_1y_2 - x_2y_1$, $b_1 = y_2 - y_3$, $b_2 = y_3 - y_1$, $b_3 = y_1 - y_2$, $c_1 = x_3 - x_2$, $c_2 = x_1 - x_3$, $c_3 = x_2 - x_1$ with usual notation.

- The elasiticity matrix for plane stress condition, $[D] = \frac{E}{1-\upsilon^2}\begin{bmatrix} 1 & \upsilon & 0 \\ \upsilon & 1 & 0 \\ 0 & 0 & \frac{1-\upsilon}{2} \end{bmatrix}$

[8.0 Marks]

Q5. a) I. Figure 5 (a) shows the control points for a Bezier curve. Sketch the curve. *(attached this sheet to your answer book)*

II. Draw the line tangent to the curve at the point u = 0.5.

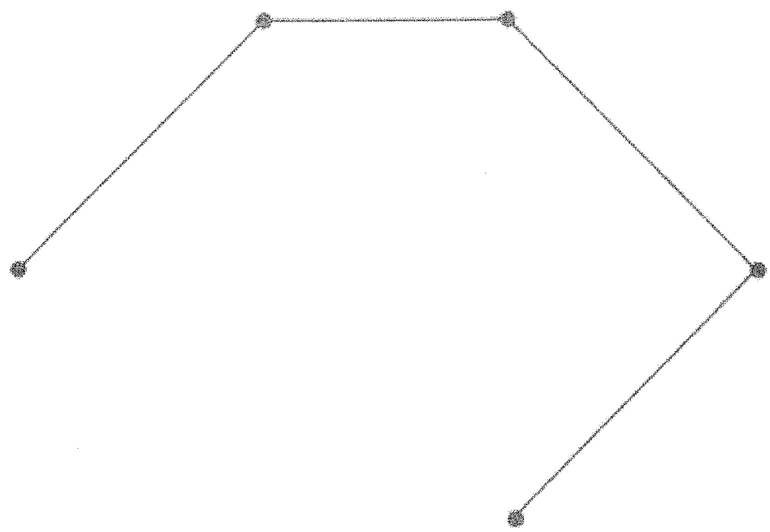

Figure 5 (a)

[2.0 Marks]

b) Figure 5 (b) shows the control points for a B-spline curve of order three (degree two). Sketch the curve.

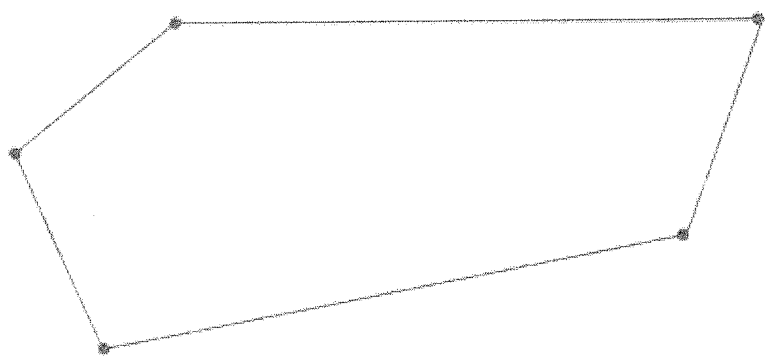

Figure 5 (b)

[1.5 Marks]

c) I. Derive the basis matrix (M) for cubic Bezier curve. Also give the corresponding blending functions.

II. What are the conditions for smoothly joining the two Bezier curve segments?

[5.5 Marks]

d) Discuss the similarities and dissimilarities of Bezier curves and B-spline curves?

[3.0 Marks]