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Classification of autoparatopisms of Latin cubes

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An action on a Latin hypercube of dimension d and order n is called a paratopism if the action is an element of the wreath product $S_n \wr S_{d+1}$. A paratopism is said to be an autoparatopism if there exists a Latin hypercube, which is mapped to itself under the action of the paratopism. Given a Latin cube (when d = 3) and for each order n, ($n \in Z^+$), up to the conjugacy in $S_n \wr S_4$ a classification is presented. It is proved that given an autoparatopism $\sigma \in S_n \wr S_{d+1}$, every conjugate of σ is an autoparatpism. The most significant consequence for the process of classification is, if $\sigma_1 = (\alpha_1,\alpha_2,\alpha_3,\alpha_4;\delta_1) \in S_n \wr S_4$ and $\sigma_2 = (\beta_1,\beta_2,\beta_3,\beta_4;\delta_2) \in S_n \wr S_4$ then, σ_1 is conjugate to σ_2 in $S_n \wr S_4$ if and only if there is a length preserving bijection η from the cycles of δ_1 to the cycles of δ_2 such that if η maps a cycle $(a_1 \ldots a_k)$ to $(b_1 \ldots b_k)$ then $\alpha_{a_1}\alpha_{a_2}\ldots\alpha_{a_k} \sim \beta_{b_1}\beta_{b_2}\ldots\beta_{b_k}$. As a consequence, it can be concluded that every autoparatopism σ_1 is conjugate to another autoparatopism σ_2 which is of one of the forms $(\alpha_1,\alpha_2,\alpha_3,\alpha_4;\varepsilon)$, $(\varepsilon,\alpha_2,\alpha_3,\alpha_4;(1\ 2))$, $(\varepsilon,\varepsilon,\alpha_3,\alpha_4;(1\ 2\ 3))$, $(\varepsilon,\varepsilon,\varepsilon,\alpha_4;(1\ 2\ 3\ 4))$ or $(\varepsilon,\varepsilon,\alpha_3,\alpha_4;(1\ 3)(2\ 4))$.

Keywords: autoparatopism, conjugates, latin cube, latin square and paratopism

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