ISSN: 1391-8796
Proceedings of 7th Ruhuna International Science & Technology Conference
University of Ruhuna, Matara, Sri Lanka
January 22, 2020

G- Frames in the quaternionic setting

Khokulan M.* and Sangarthas M.

Department of Mathematics and Statistics, University of Jaffna, Sri Lanka

Quaternions are an extension of complex numbers with one real and three imaginary parts. Quaternionic Hilbert space is a vector space under multiplication by quaternionic scalars, from the non-commutativity, the quaternionic Hilbert spaces are defined in two ways: left/right quaternionic Hilbert spaces. Frame is a spanning set of vectors, which are generally over complete (redundant) in a quaternionic Hilbert space. G- frames are natural generalization of frames and provide more choices on analyzing functions from frame expansion coefficients. In this research, the construction of G-frame is reported and relation between G-frame and canonical dual G-frame is established. Let $U_{\mathbb{H}}^{L}$ and $V_{\mathbb{H}}^{L}$ be left quaternionic Hilbert spaces and $\{\mathcal{V}_k : k \in \mathbb{I}\} \subseteq V_{\mathbb{H}}^{L}$ is a sequence of quaternionic Hilbert spaces. Let $\mathfrak{B}(U_{\mathbb{H}}^{L}, \mathcal{V}_k)$ be the collection of all bounded linear operators from $U_{\mathbb{H}}^{L}$ into$\mathcal{V}_k$. A family $\{\Gamma_k \in \mathfrak{B}(U_{\mathbb{H}}^{L}, \mathcal{V}_k) \ : k \in \mathbb{I}\}$ is called a generalized frame or simply G-frame for $U_{\mathbb{H}}^{L}$ with respect to $\{\mathcal{V}_k : k \in \mathbb{I}\}$ if there exist constants $0 < C \leq D <\propto$ such that $C\|\phi\|^2 \leq \sum_{k\in\mathbb{I}}\|\Gamma_k\phi\|^2 \leq D\|\phi\|^2$, for all $\phi \in U_{\mathbb{H}}^{L}$, where C and D are G-frame bounds. G-frame operator F_g can be defined as $F_g\phi = \sum_{k\in\mathbb{I}} {\Gamma_k}^{\dagger}\, \Gamma_k\phi$, for all $\phi \in U_{\mathbb{H}}^{L}$, where ${\Gamma_k}^{\dagger}$ is the adjoint operator of Γ_k. Frame operator F_g is self adjoint, bounded and invertible. If $\{\Gamma_k \ : k \in \mathbb{I}\}$ be a G-frame for $U_{\mathbb{H}}^{L}$ with respect to $\{\mathcal{V}_k : k \in \mathbb{I}\}$ and $\widetilde{\Gamma_k} = \Gamma_k {F_g}^{-1}$, then $\{\widetilde{\Gamma_k} \ : k \in \mathbb{I}\}$ is a G-frame for $U_{\mathbb{H}}^{L}$ with frame bounds $\frac{1}{D}$ and $\frac{1}{C}$. We call it the canonical dual G-frame of $\{\Gamma_k \ : k \in \mathbb{I}\}$. Finally, we conclude that $\{\Gamma_k \ : k \in \mathbb{I}\}$ and $\{\widetilde{\Gamma_k} \ : k \in \mathbb{I}\}$ are dual G-frames with respect to each other.

Keywords: frames, G-frames, quaternion and quaternionic Hilbert spaces

*Corresponding author: mkhokulan@gmail.com