

**University of Ruhuna- Faculty of Technology**  
**Bachelor of Engineering Technology and Information & Communication Technology**  
**Level 1 (Semester 1) Examination, October 2018**

**Course Unit: TMS 1113 Foundation of Mathematics**  
**Time Allowed 3 hours**

Answer all **Six (06)** questions. Calculators are allowed to use for calculations.

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All symbols have their usual meaning  
[Any common values Ex: Acceleration of gravity,  $g = 9.8 \text{ ms}^{-2}$ ]

1. 1.1 If  $f(x) = \frac{x}{x^2-16}$  and  $g(x) = \frac{7}{x^2+5x+4}$  find the followings.

- i.  $f(x) + g(x)$
- ii.  $f(x) - g(x)$
- iii.  $f(x) \times g(x)$
- iv.  $f(x)/g(x)$

1.2 When getting into shape by exercising, the subject's maximum recommended number of heart beats per minute (h) can be determined by subtracting the subject's age (a) from 220 and then taking 75% of that value. Expressed the relationship for (h) using age (a).

1.3 Solve the following equations.

- i.  $x^2 = 49$
- ii.  $2\sqrt{a} = 24$
- iii.  $\sqrt{4x+9} = -13$
- iv.  $\frac{28}{\sqrt{5x+1}} = -7$

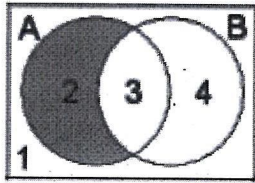
1.4 Solve the following equations using the quadratic formula. Reduce answers to their simplest form or to the simplest radical form.

- i.  $x^2 + 2x - 8 = 0$
- ii.  $6x^2 + 13x - 28 = 0$
- iii.  $14x^2 = 12x + 32$
- iv.  $\frac{1}{3}x^2 + \frac{3}{4}x - 3 = 0$

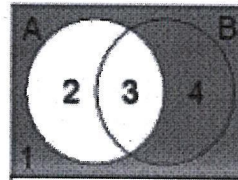
1.5 A reservoir can be filled by an inlet pipe in 24 hours and emptied by an outlet pipe in 28 hours. The foreman starts to fill the reservoir, but he forgets to close the outlet pipe. Six hours later he remembered and closes the outlet. How long does it take to fill the reservoir?

2. 2.1 Refer to the diagram to answer the questions below. What set notation would you use to represent the following shaded regions?

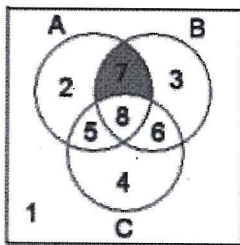
i) Only Region 2 is shaded



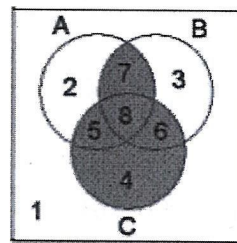
ii) Regions 1 and 4 are shaded



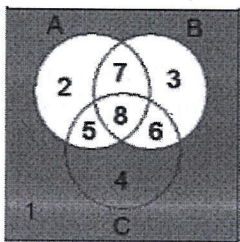
iii) Only Region 7 is shaded



iv) Regions 4, 5, 6, 7 and 8 are shaded



v) Regions 1 and 4 are shaded



2.2 In a survey of 130 people, the following data were collected: 106 people subscribed to the newspaper, 29 people subscribed to magazines, and 17 people were members of a mail CD club. Seventeen subscribed to both the newspaper and the magazines, 5 people subscribed to magazines and were members of a CD club, and 10 people subscribed to the newspaper and were members of a mail CD club. Three people subscribed to both the newspaper and magazines and were members of a mail CD club.

- i. Make and fill in a Venn diagram to illustrate this situation.
- ii. Find the number of people who subscribed only for newspapers.
- iii. Find number of people who did not subscribe for newspaper, magazines or mail CD club.

2.3 i. Write following in set builder notation.

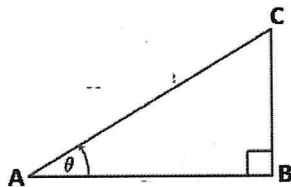
{January, February, March, April, May}

ii. Write out the set  $\{x: x \text{ is an integer less than } 4\}$  in roster notation.

2.4 If  $U = \{1,2,3,4,5,6\}$ ,  $S = \{2,4,6\}$ ,  $T = \{1,2,4\}$  and  $V = \{4,5,6\}$  find the followings.

- i.  $S \cup (T \cap V)$
- ii.  $(S \cup T) \cap V$
- iii.  $(S \cup T)'$
- iv.  $(V \cup S)'$
- v.  $S \cap (V \cap T)'$
- vi.  $(S' \cap V') \cup T$
- vii.  $(S' \cup V') \cap T$
- viii.  $(S' \cup T) \cap V'$
- ix.  $T \cup V' \cup S'$
- x.  $T \cup V' \cap S'$
- xi.  $(V \cap T)' \cup S$
- xii.  $V \cup (S \cap T)'$

3. 3.1 If  $\hat{CAB} = \theta$ , define the Cosec  $\theta$ , Sec  $\theta$  and Cot  $\theta$  for following ABC, right-angled triangle.



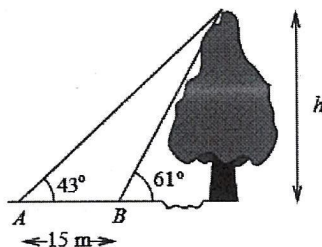
3.2 In question 3.1, if point A lies at co-ordinate (2, 3), point B at (8, 3) and point C at (8, 7). Determine the followings.

- i. the distance AC
- ii. the gradient of the straight line AC.
- iii. the values for cosec  $\theta$ , sec  $\theta$  and cot  $\theta$

3.3 By proving the following equation and calculate the value of  $\cot 15^\circ$ .

$$\tan \frac{\alpha}{2} = \pm \sqrt{\frac{1 - \cos \alpha}{1 + \cos \alpha}}$$

3.4 A forestry worker wishing to establish the height  $h$  of a tall tree on the other side of a river bank measures the angle of elevation of the top of the tree to be  $61^\circ$  at A, but  $43^\circ$  at a point B 15 m from A. Find the height of the tree.



3.5 i. Perform the following calculations

1.  $72^\circ 38' + 36^\circ 46'$

2.  $67^\circ - 24^\circ 19'$

ii. Express the angles  $0^\circ$  and  $10^\circ$  in terms of radians.

iii. Write the angles  $2\pi/3$  and  $\pi/4$  in terms of degrees.

3.6 Find the value of  $\sqrt{3} \operatorname{cosec} 20^\circ - \sec 20^\circ$

4. 4.1 If  $A = \begin{bmatrix} 3 & 2 & 1 \\ 0 & 1 & -2 \\ 1 & 3 & 3 \end{bmatrix}$ , Compute the determinant of the matrix A, by expansion about

i. the first row

ii. the first column.

4.2 Let  $A = \begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix}$ ,  $B = \begin{bmatrix} 4 & 0 & 1 \\ 0 & 2 & 1 \end{bmatrix}$  and  $D = [1 \ 2]$ . Compute the followings.

i.  $(DB)^T$

ii.  $BA + D^T$

4.3 Compute the inverse of following matrix Z.

$$P = \begin{bmatrix} 4 & 8 & 1 \\ 2 & -3 & 2 \\ 1 & 7 & -3 \end{bmatrix}$$

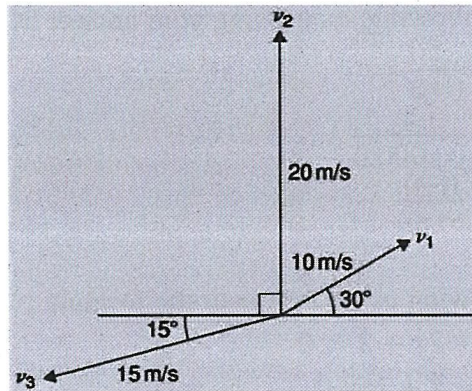
4.4 Using the above (Q.4.3) calculated inverse matrix ( $P^{-1}$ ) solve the following sets of equations.

$$4x + 8y + z = -6$$

$$2x - 3y + 2z = 0$$

$$x + 7y - 3z = -8$$

5. 5.1 Velocities of  $10 \text{ ms}^{-1}$ ,  $20 \text{ ms}^{-1}$  and  $15 \text{ ms}^{-1}$  act as shown in following Figure. Calculate the magnitude of the resultant velocity and its direction relative to the horizontal.



- 5.2 Given that  $\vec{p} = 3\mathbf{i} + 2\mathbf{k}$ ,  $\vec{q} = 4\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}$  and  $\vec{r} = -3\mathbf{i} + 5\mathbf{j} - 4\mathbf{k}$ . Determine the followings.

- i.  $-\vec{r}$
- ii.  $3\vec{p}$
- iii.  $2\vec{p} + 3\vec{q}$
- iv.  $-\vec{p} + 2\vec{r}$
- v.  $0.2\vec{p} + 0.6\vec{q} - 3.2\vec{r}$

- 5.3 Show that  $\frac{\mathbf{i} - \mathbf{j}}{\sqrt{2}}$  is a unit vector.

- 5.4 For vectors  $\vec{a} = -7\mathbf{i} + 4\mathbf{j} + \frac{1}{2}\mathbf{k}$  and  $\vec{b} = 6\mathbf{i} - 5\mathbf{j} - \mathbf{k}$  find

- i.  $\vec{a} \cdot \vec{b}$
- ii.  $\vec{a} \times \vec{b}$
- iii.  $|\vec{a} \times \vec{b}|$
- iv.  $\vec{b} \times \vec{a}$
- v. The angle between the vectors

- 5.5 Forces of  $(\mathbf{i} + 3\mathbf{j})$ ,  $(-2\mathbf{i} - \mathbf{j})$ ,  $(\mathbf{i} - 2\mathbf{j})$  newtons act at three points having position vectors of  $(2\mathbf{i} + 5\mathbf{j})$ ,  $4\mathbf{j}$  and  $(-\mathbf{i} + \mathbf{j})$  meters, respectively. Calculate the magnitude of the moment. Note that, *Moment* = *Force*  $\times$  *distance*

6. 6.1 Evaluate the following, expressing your answer in Cartesian form  $(a + bi)$ , where  $a$  and  $b$  are real numbers and  $i^2 = -1$ .

- i.  $(1 + 2i)(4 - 6i)^2$
- ii.  $(1 - 3i)^3$

- iii.  $i(1 + 7i) - 3i(4 + 2i)$
- iv.  $\{(7 + 9i)^2 - (1 + 5i - 4i^2)\}^2$

6.2 Evaluate the following, expressing your answer in Cartesian form ( $a + bi$ ):

i.  $\frac{i}{(1+i)}$

ii.  $\frac{2}{(1-i)(3+i)}$

iii.  $\frac{(i-4)}{(2i-3)}$

6.3 Solve the following using the quadratic formula.

i.  $z^2 + 2z + 2 = 0$

ii.  $z^2 - z + 1 = 0$

iii.  $2z^2 - 2iz - 5 = 0$

6.4 Find the modulus and the argument of the following complex numbers.

i.  $z = 5 + 3i$

ii.  $p = -7i$

iii.  $q = 4.4$

iv.  $w = \frac{(3i-9)}{(2i-1)}$

6.5 Write down the above 6.4 question's complex numbers into polar form.