University of Ruhuna- Faculty of Technology

Bachelor of Engineering Technology

Level 1 (Semester 1) Examination, October 2018

Course Unit: TMS 1143 Physics of Mechanical Systems

Time Allowed 3 hours

Answer all Six (06) questions

All symbols have their usual meaning. $g = 9.81\ \text{m/s}^2$

Briefer answers are anticipated whenever possible.

1. (i) Initially, an object is at a position with coordinates: $(x_1, y_1) = (3.0\ \text{m}, 0\ \text{m})$ on the ground. Then the object is moved to a new position with coordinates $(x_2, y_2) = (4.0\ \text{m}, -1.0\ \text{m})$.

   (a) Find the displacement of the object in unit-vector notation.

   (b) Find the angle of the displacement relative to the positive x direction.

   (ii) The coordinates of an object moving in the xy plane is specified by,

   $$(x, y) = [(4.00 + 1.00t^3), (-5.00t + 3.00t^4)],$$

   where t is in seconds and x and y coordinates are in meters.

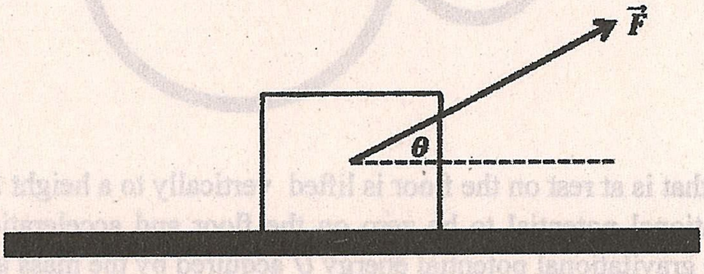
   In unit-vector notation,

   (a) Calculate the velocity $\vec{v}$ of the object at $t = 1.00$ s.

   (b) Calculate the acceleration $\vec{a}$ of the object at time t.

   (iii) At the beginning a block of mass m stays stationary on the ground. Then an upward force $\vec{F}$ that makes an angle θ with the horizontal direction and that has a magnitude F is applied on the block as shown in the figure below. If the block moves without lifting and the coefficient of kinetic friction between the block and the ground is μ_k then, show that the acceleration of the block can be written as,

   $$a = \frac{F}{m}(\cos\theta + \mu_k \sin\theta) - \mu_k g.$$



2. (i) A horizontal tabletop is 1.00 m high from the floor level. A small ball rolls off from the table edge and strikes the floor at a point 1.50 m horizontally from the edge. Find its speed at the instant it leaves the table. [*Some common particle-motion equations: $v = v_0 + at$, $x - x_0 = v_0 t + \frac{1}{2}a t^2$, $v^2 = v_0^2 + 2a(x - x_0)$].

   (ii) (a) Write down an expression for the magnitude of the centripetal acceleration of an object moving in a uniform circular path of radius r with a velocity v.

(b) A satellite performs uniform circular motion in a circular orbit 1000 km above Earth's surface. It has a period of 100.0 minutes. If the Earth's radius is $R_E = 6.37 \times 10^6$ m then, find the magnitude of the centripetal acceleration of the satellite.

(iii) In a carnival ground, a fully loaded roller-coaster car with passengers has a total mass of 1000 kg. Assume that the speed of the car does not change as it passes over the top of a circular hill of radius 20 m with a speed of 10 m/s. Find the magnitude of the normal force $\vec{F}_N$ on the car at the top, and the direction of the normal force.

3. (i) Initially, a disk rotates with a constant angular speed of 100 rad/s. Then it is slowed down at a constant angular deceleration of magnitude 4.0 rad/s^2. Using the appropriate kinematic equations for rotation,

(a) Find the time that the disk takes to come to a stop.

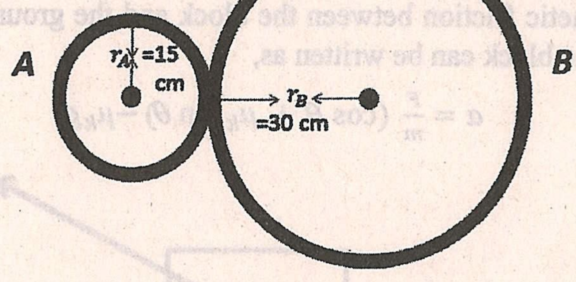
(b) Find the angle that the disk rotates during that time.

(ii) A wheel A of radius $r_A = 15$ cm is in contact with a wheel B of radius $r_B = 30$ cm as shown in the figure below. From rest (at $t = 0$) the angular speed of wheel A is increased at a constant angular acceleration of magnitude $\alpha_A = 2.0$ rad/s^2. Assuming the wheels do not slip at the contact point then,

(a) Find the magitude of the angular acceleration of the wheel B, using the relations between linear and angular variabales.

(b) Find the angular speed of wheel B at time $t = 13$ s, in revolutions per second.

(iii) One end of a 1.00 m long massless rod is hung from a pivot and at the other end a small ball of mass 2.00 kg is attached to it resembling a pendulum. Write down an expression for the magnitude of the torque, and thus when the pendulum makes a 30° angle from the vertical direction, find the magnitude of the gravitational torque about the pivot.



4. (i) A mass m that is at rest on the floor is lifted vertically to a height h. If you assume that the gravitational potential to be zero on the floor and acceleration of gravity is g then, what is the gravitational potential energy U acquired by the mass at the height h.

(ii) For an isolated system that includes only mechanical energies show that, $K_f + U_f = K_i + U_i$. Here K and U represents the kinetic and potential energies, and subscripts i and f represents the initial and final state of the system respectively.

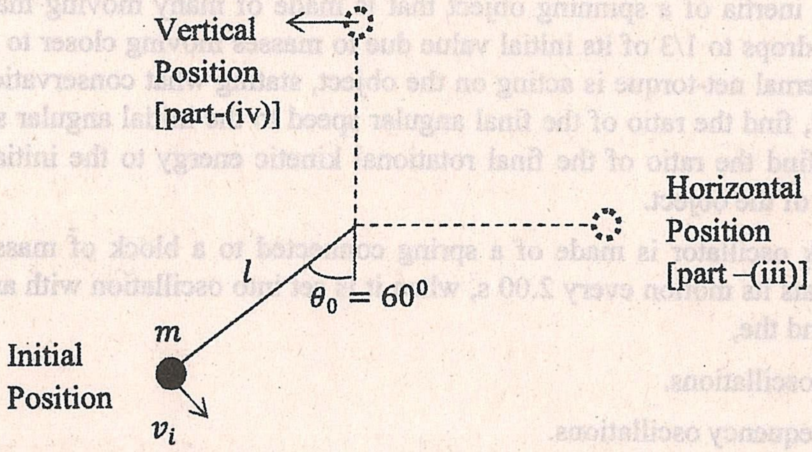
(iii) A pendulum of an effective length l is made of a massless string and a bob of mass m as shown in the figure below (in next page). Initially, when the string creates an angle of 60° with the vertical direction the speed of the bob is v_i. The acceleration of gravity is g

and you can neglect the air resistance. If the pendulum swings down and then up to a horizontal position, show that the least value of the initial speed can be written as, $v_i = \sqrt{gl}$. (Note: $\cos 60^o = 0.5$)

(iv) If the pendulum swings down and then up to a vertical position, while keeping the string straight [Instead of a horizontal position like in part-(iii)], then,

(a) Show that the least value of the final speed of the bob can be written as, $v_f = \sqrt{gl}$.

(b) Thus, show that the least value of v_i can be written as, $v_i = 2\sqrt{gl}$.



5. (i) Write down an expression for the elastic potential energy U stored in a spring in terms of the spring constant k and the extension (or compression) of the spring x.

(ii) Using the expression in part-(i) show that the spring force can be written as, $F = -kx$.

(iii) For an isolated system that includes only mechanical and thermal energy, show that the change in the thermal energy is given by, $\Delta E_{th} = -\Delta U - \Delta K$.

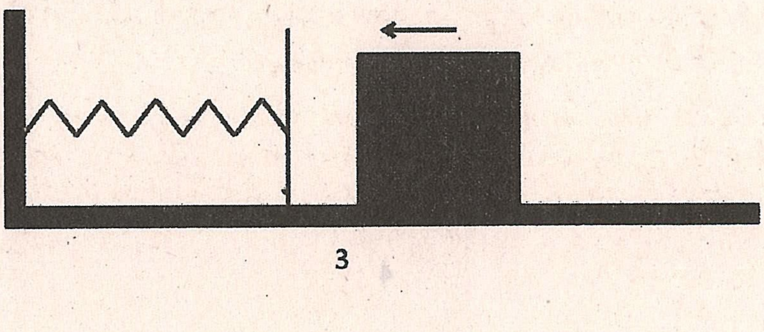
(iv) A block of mass m = 2.0 kg slides head on to a spring as shown in the figure below. The spring constant of the spring is k = 200 N/m. Just as the block reaches the uncompressed spring it's speed is v_i = 0.80 m/s (i.e. the speed just before colliding). Then the block is brought to a rest by the spring while the block and spring are in contact. When the block stops, it compresses the spring by 5 cm.

(a) Find the work done W_s due to compression by the spring force.

(b) Find the increase in the elastic potential energy ΔU in the spring due to compression.

(c) Find the increase in thermal energy ΔE_{th} of the block-spring system [hint: part-(iii)].

(d) Find the coefficient of kinetic friction between the block and the floor [you may use the expression from part-(iii)].



6. (i) (a) At the center of mass (c.o.m.) the moment of inertia about its perperpendicular axis of a thin rod of mass M and length L is , $I_{\text{com}} = \frac{1}{12}ML^2$. Thus, using an appropriate theorem, show that the moment of inertia at one end of the rod about its perperpendicular axis is given by, $I_{end} = \frac{1}{3}ML^2$.

(b) A thin rod of length L has one end on the ground. It is allowed to fall after holding vertically at rest (i.e. $v_i = 0$). Stating what conservation principle can be applied and assuming that the end on the floor does not slip, show that the speed of the other end of the rod just before it hits the ground can be written as, $v_f = \sqrt{3gL}$.

(ii) The rotational inertia of a spinning object that is made of many moving masses (e.g. a spinning star) drops to 1/3 of its initial value due to masses moving closer to the rotation axis. If no external net-torque is acting on the object, stating what conservation principle can be applied, find the ratio of the final angular speed to the initial angular speed of the object. Thus, find the ratio of the final rotational kinetic energy to the initial rotational kinetic energy of the object.

(iii) A spring-block oscillator is made of a spring connected to a block of mass 1 kg. The oscillator repeats its motion every 2.00 s, when it is set into oscillation with an amplitude of 10.0 cm. Find the,

(a) period of oscillations.

(b) angular frequency oscillations.

(c) spring constant of the spring.

(d) maximum speed of the mass.