# University of Ruhuna

## B.Sc.(General) Degree

## Level I (Semester I) Examination - December 2020

**Subject: Applied Mathematics**
**Course Unit: AMT112$\beta$**
**(Mathematical Foundation of Computer Science)** **Time: Two (02) Hours**

### Answer All Questions

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**1.** a) Show that, $p \leftrightarrow \sim q$ and $p \rightarrow q$ are not logically equivalent using a **truth table**. *[10]*

b) Show that $\sim (p \vee (\sim p \wedge q)) \equiv (\sim p \wedge \sim q)$ using **laws of propositions**. *[20]*

c) Write the **negation** of "For a real number $x$, if $x > 3$ then $x^2 > 9$". *[05]*

d) Write the **inverse** of "If today is Sunday, then it is a holiday". *[05]*

e) Using **pattern proof**, show that $t$ is a valid conclusion from the premises; $p \rightarrow q$, $q \rightarrow r$, $r \rightarrow s$, $\sim s$, $p \vee t$ *[20]*

f) Consider the following;

"All people are responsible. Anyone who is responsible and wear masks can free from Covid-19. Anyone who is free from Covid-19 will be happy. Sarath is a person wearing masks".

(i) Change the above facts into predicate forms. *[10]*

(ii) Translate above predicate forms into clauses. *[10]*

(iii) Using **Resolution Principle** prove that, "Sarath will be happy". *[20]*

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**2.** a) Simplify 11110111.1 / 1111 *[10]*

b) Using **computer method**, convert $11110.011_2$ into decimal. *[15]*

c) A computer has a word length of 8 bits and uses two's complement method for calculations. Translate -109 into the number format used by the above computer. *[15]*

d) Using **two's complement**, perform the followings;

(i) $17 - 73$ in a 8 bits word length computer. *[20]*

(ii) 119/7 *[40]*

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**3.** a) For all $A$, $B$ in a Boolean Algebra ß, prove $A + (A.B) = A$ *[10]*

b) Find the **dual** of the Boolean expression $A + (AA') + (B.A) = 1$ *[10]*

c) Define;

(i) a fundamental product, *[05]*

(ii) a sum of product expression, *[05]*

(iii) a complete sum of product form. *[05]*



**Continued.**

d) (i) Sketch the logic circuit for the Boolean expression $E(x, y, z) = z(x' + y) + y'$ *[10]*

(ii) Find the **complete sum-of-product form** of the above Boolean expression. *[15]*

e) Use **Karnaugh maps** to simplify each of the following Boolean expression;

(i) $A'B'C' + A'B'C + A'BC + A'BC' + AB'C$ *[20]*

(ii) $A'B'C' + A'BC' + ABC' + AB'C$ *[20]*

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**4.** a) Prove the correctness of the following Hoare Triple using **assignment rule**;

$\{y > 0\}\{x = y + 3\}\{x > 3\}$ *[10]*

b) Prove the correctness of the following using **loop rule**; *[20]*

```
i = 0; y = 1;
if x ≤ 0 then return "error"
else
{
 while(y < x){
      i = i + 1;
      y = 2 * y;
      }
}
return i
```

c) Using **direct proof**, show that if $p$ and $q$ are both perfect squares, then $pq$ is also a perfect square. *[10]*

d) Using **contrapositive proof**, show that if $m = ab$ where $a$ and $b$ are positive integers, then $a \leq \sqrt{m}$ or $b \leq \sqrt{m}$. *[20]*

e) A person deposits Rs. 10000 in a savings bank account at a bank yielding 9% per year with interest compounded annually.

(i) Find a reccurence relation which satisfies the amount in the account after n years. *[10]*

(ii) Use expand guess and verify method to find the closed-form solution of the recurrence relation you have found above. *[20]*

f) The first 4 numbers of a sequence are given by $7, 14, 23$ and $34$. Find a recurrence formula of the form $S(n) = S(n-1) + g(n)$ for the $n^{th}$ term in the sequence. *[10]*

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