UNIVERSITY OF RUHUNA
DEPARTMENT OF MATHEMATICS
BACHELOR OF SCIENCE (GENERAL) DEGREE
LEVEL I (SEMESTER I) EXAMINATION-DECEMBER 2020

Subject : Mathematics
Course Unit : MAT113δ - Introductory Statistics
Time: One (01) Hour

Answer All Questions

1. (a) Let X be a discrete random variable with the following probability mass function

$$P(X = x) = \begin{cases} kx, & \text{for } x = 1, 2, 5 \\ 0, & \text{Otherwise.} \end{cases}$$

(i) Find the constant k.
(ii) Find the expected value of X.
(iii) Find the expected value of $2X + 3$.

(25 marks)

(b) The probability that a patient recovers from a rare blood disease is 0.4. Suppose 150 people are known to have infected this disease. Define random variable X as the number of patients recovers from that rare blood disease.

(i) What is the distribution of X?
(ii) Find expectation and variance of X.
(iii) Can you apply normal approximation to find probabilities? Justify your answer.
(iv) What is the probability that less than 50 survive?

(45 marks)

(c) Suppose that monthly suicide rate in a certain country is 1 per 100,000 people. Give an approximation to the probability that in a city of 400,000 in this country there will be no more than 2 suicides in the next month? **(30 marks)**

2. (a) The cumulative distribution function of a random variable X is

$$F(X) = \begin{cases} 1 - e^{-2x}, & x \geq 0 \\ 0, & x < 0 \end{cases}$$

(i) Find $Pr(-3 < X < 4)$.

(ii) Find the probability density function, $f(x)$ of X.

(iii) Find $Pr\left(\left|X - \frac{1}{2}\right| \geq 1\right)$

(iv) Show that the moment generating function of the random variable X is given by

$$M_X(t) = \frac{2}{2-t}, \qquad t < 2.$$

Hence find the expectation and the variance of X.

(v) A non-negative random variable is said to be memoryless if

$$P(X > s + t | X > t) = P(X > s) \quad \text{for all} \quad s, t \geq 0$$

Show that the random variable X is memoryless.

(85 Marks)

(b) It is known that the number of items produced in a factory during a week is a random variable with mean 50 and variance 25. What is the fraction of this week production will be between at least 40 and at most 60? **(15 Marks)**

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