University of Ruhuna

B.Sc.(General) Degree

Level II (Semester I) Examination - January - 2022

Subject: Applied Mathematics/Industrial Mathematics
Course Unit: AMT211β/IMT211β
Classical Mechanics III (Fluid Dynamics) **Time: Two (02) Hours**

Answer All Questions.

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1. a) For all p, q and r,

$$\underline{v} = (x + 3py + 2z)\underline{i} + (x + y - qz)\underline{j} + (rx - 2y + z)\underline{k}$$

is the velocity for possible motion of an incompressible fluid. If this motion is irrotational, find

(i) the values of p, q and r,

(ii) the velocity potential. [50 marks]

b) Find the equation of pathlines for a fluid flow with the velocity, $\underline{u} = x(3t + 1)\underline{i} + 2y\underline{j}$ for $x > 0$, $y > 0$ and t is time. [30 marks]

c) Suppose the velocity of a fluid flow is given by,

$$\underline{u} = -\frac{ky}{x^2 + y^2}\underline{i} + \frac{kx}{x^2 + y^2}\underline{j} + 2xy\underline{k} \text{ ; where } k \text{ is a constant.}$$

Show that the above represents a possible motion of an incompressible flow. [20 marks]

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2. a) A long tube of length l has a slowly decreasing circular cross section. It is inclined at an angle θ to the horizontal with its smaller cross-section downwards and water flows steadily through it from the upper to the lower end. The radius of the tube at its upper end is 1.5 times that of the lower end. At the lower end, the water is delivered at atmospheric pressure π. If the pumping pressure is 3 times the atmospheric pressure, find the speed U of the fluid when it leaves the tube (Assume the density of water is given by ρ and g is referred to as the acceleration of gravity). [30 marks]

b) A fluid with density ρ is contained in the region bounded by two concentric rigid spherical surfaces of radius $R_1, R_2 (R_2 > R_1)$. The fluid is initially at rest. Now, the inner surface is given a sudden velocity $q\underline{i}$ where $\underline{i}$ is a constant unit vector. Assuming, in the usual notation that for the motion setup at a point having spherical polar co-ordinates (r, θ, φ) in the fluid, the velocity potential is given by,

$$\phi(r, \theta) = (Ar + Br^{-2})\cos\theta; \quad (R_1 \leq r \leq R_2),$$

where A and B are constants. If ρ is a constant and motion is irrotational, find the velocity potential. [40 marks]

Continued.

c) A source and a sink of equal strength m are situated on the x axis distance c and $-c$ apart respectively from the origin 0.

(i) Find the resulting velocity potential $\phi(r,\theta)$ at $P(r,\theta)$ in spherical polar co-ordinates under axial symmetry.

(ii) Using the above system, show that the velocity potential at $P(r,\theta)$ due to the doublet of strength μ places at the origin and along the axis $\theta = 0$ is $\frac{\mu\cos\theta}{r^2}$. [30 marks]

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3. a) In the usual notation, suppose that $\varphi(x,y)$ is the stream function for the velocity field $\underline{u} = u_1\underline{i} + u_2\underline{j}$.
Show that $\underline{\nabla}\varphi$, is perpendicular to $\underline{u}$. [20 marks]

b) Considering a flow with velocity field $\underline{u} = y\underline{i} - (x-2t)\underline{j}$,

(i) find the stream function $\varphi(x,y)$,

(ii) find the equation of streamlines,

(iii) sketch the streamlines you obtain in part (ii) above for $t = 1$. Hence, sketch the equipotential lines using the same figure. [50 marks]

c) Obtain the velocity potential and stream function for each of the following two dimensional irrotational flows,

(i) $w(z) = lnz$

(i) $w(z) = \frac{1}{z+1}$ [30 marks]

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4. a) In the usual notation, state the Milne-Thomson Circle theorem. [15 marks]

b) Let a long infinite cylinder of radius a be placed in a uniform stream having a velocity -$U\underline{i}$. Find

(i) the complex velocity potential,

(i) the stagnation points. [25 marks]

c) A doublet of strength μ is placed at the point $Z(c,0)$ where $c > a$, with its axis in the direction which makes an angle θ with the positive real axis.
Use the circle theorem to find an equivalent system for the above system. [60 marks]

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