UNIVERSITY OF RUHUNA

Faculty of Engineering

Mid-Semester 5 Examination in Engineering: June 2015

Module Number: IS5310 **Module Name: Complex Analysis and Mathematical Methods**

[Two hours]

[Answer **all questions,** each question carries five marks]

Q1. Define the continuity and differentiability of a complex function $f: D \to C,\ D \subset C$ at a point $z_0 \in D$. [01 Mark]

Show that if the complex function f is differentiable at $z = a$ then f is continuous at $z = a$. [0.5 Marks]

a) Show that

$$\lim_{z \to 0} \frac{\bar{z}}{z}$$

does not exist. [01 Mark]

b) Using the definition, find the derivative of

$$f(z) = \frac{1}{z}$$

[01 Mark]

c) Write the function $f(z) = 3z^3 + z - 2$ in terms of real and imaginary parts $u(x, y)$ and $v(x, y)$ such that $f(z) = u(x, y) + i\, v(x, y)$. Verify that the real and imaginary parts satisfy Cauchy-Riemann equations. [1.5 Marks]

Q2. a) Let $f(z) = \sqrt{xy}$. Show that $f'(0)$ does not exist and yet Cauchy-Riemann equations are satisfied at $z = 0$. [2.5 Marks]

b) Express the function $f(z) = z^3$ in plane polar coordinates (r, θ) where $r > 0$ and $0 < \theta < 2\pi$.

i) Show that $f(z)$ satisfies Cauchy-Riemann equations in polar form.
ii) Hence, find $f'(z)$. [2.5 Marks]

Q3. Let $f(z)$ be an analytic function such that $f(z) = u(x, y) + i\, v(x, y)$ where the real part of $f(z)$ is given by $u(x, y) = e^x\,(x \cos y - y \sin y)$.

a) Show that $u(x, y)$ is a harmonic function. [02 Marks]

b) Find a harmonic conjugate $v(x, y)$ of $u(x, y)$. [02 Marks]

c) Find the analytic function $f(z)$ in terms of z. [01 Mark]

Q4. Let $z(t) = x(t) + i\,y(t)$, $a \le t \le b$ be a piecewise smooth curve (contour) C, and $z'(t) = x'(t) + i\,y'(t)$, where $'$ denotes differentiation with respect to t. Then show that

$$\int_C f(z)dz = \int_a^b f\big(z(t)\big)\, z'(t)dt$$

[01 Mark]

Evaluate the integral

$$\int_C z^2\, dz,$$

when C is given by

a)

$$z(t) = \begin{cases} 2t & ; 0 \le t \le 1 \\ 2 + i(t-1) & ; 1 \le t \le 2 \end{cases}$$

[02 Marks]

b) the straight line segment joining (1,1) to the point (3,6) on the complex plane

[02 Marks]