# UNIVERSITY OF RUHUNA

**Faculty of Engineering**

**Semester 5 Examination in Engineering: March 2022**

**Module Number: CE5253** **Module Name: Uncertainty in Engineering Measurements (C-18)**

[Three Hours]

[Answer all questions. Each question carries **FIFTEEN** marks]
All Standard Notations denote their regular meanings

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Q1 a) An angle was measured by two different observers using the same instrument, as given in Table Q1-1.
Calculate:
i. The standard deviation of each observer's readings.
ii. The standard error of the arithmetic means.
iii. The most probable value (MPV) of the angle.

[6.0 Marks]

b) Measured angles surrounding a station "O" are as given below.

| | | | | | |
|---|---|---|---|---|---|
| a | = | 75° | 16' | 10'' | wt. 1 |
| b | = | 80° | 38' | 27'' | wt. 2 |
| c | = | 150° | 28' | 15'' | wt 1 |
| d | = | 53° | 37' | 13'' | wt 1 |
| a+b | = | 155° | 54' | 45'' | wt. 2 |
| c+d | = | 204° | 05' | 30'' | wt. 2 |

Starting from fundamentals, find the most probable values of a, b, c, and d using the direct method (Method of correlatives) of least squares.

[9.0 Marks]

Q2 a) Mean traffic flow in roads A and B both is 350 vehicles per hour. But the standard deviation is 21 on road A and 150 in Road B. Describe the difference in the traffic flow of road A and B.

[2.0 Marks]

b) A regression model has been developed to predict the monthly expenditure of university students. The regression model yielded $y\hat{}$ = 5000 + 0.15x. Where, x is the monthly household income.

i) Kamal is a university student and his household income is 100,000 LKR per month. Predict Kamal's monthly expenditure using the regression model.

[1.0 Mark]

ii) Kamal declares that his monthly expenditure is 17000 LKR. Calculate the error in prediction regarding Kamal.

[1.0 Mark]

c) The linear regression line for traffic flow in road A is Us=150(1-k/20), while it is Us=90(1-k/20) for road B. Interpret the relationship between flow and mean speed for these two roads and discuss the characteristics of traffic flow in these two roads.
Us - Space mean speed and k – Traffic Density

[4.0 Marks]

d) $R^2$ values of the models fitted for these models are 0.5 and 0.8 for roads A and B respectively. Explain the meaning of these values in terms of the accuracy of the models.

[3.0 Marks]

e) One part of a given road is damaged and hence there is a difficulty in traffic movement in that part. It was observed that the time to pass this damaged part is increased with the length of the vehicle. Draw the shape of a linear regression, which can depict the relationship between time and the length of the vehicle. [Explain your decision-making procedure and name the axes of your graph.]

[4.0 Marks]

Q3. The data collected during a vehicle speed distribution study is given in Table Q 3- 1. Determine the following statistical parameters related to vehicle speed referring to the Z table given in Table Q3-2 and useful equations given at the end of the paper.

a) i. Mean speed

[2.0 Marks]

ii. Standard deviation

[2.0 Marks]

iii. Standard error

[2.0 Marks]

iv. 95% confidence interval (two tail)

[2.0 Marks]

b) If $f(t) = texp^{(-t^2/2)}$ when $t \geq 0$ and $f(t) = 0$ when $t < 0$, compute the mean and the variance of $x$.

[5.0 Marks]

c) Determine the mean and the variance of a random variable x of which probability function is given by

$$f(x) = \begin{cases} 1/3 & x = 0 \\ 1/2 & x = 1 \\ 1/6 & x = 2 \end{cases}$$

[2.0 Marks]

Q4 a) Compute the Cov[$x$, $y$] and $\rho(x, y)$ for the random variables, $x$ and $y$ given by the joint density function

$$f(x,y) = \begin{cases} 1/4 & x = 1, & y = 2 \\ 1/4 & x = 2, & y = 3 \\ 1/2 & x = 3, & y = 4 \end{cases}$$

[6.0 Marks]

b) The number of load repetitions to induce the fatigue cracking of a certain hot mix asphalt is given by

$$N_f = f_1(\varepsilon_t) - f_2$$

For this problem, $f_1 = 0.00462$ with a coefficient of variation of 30%, $f_2 = 2.69$ with a coefficient of variation of 5%, the coefficient of correlation for $f_1$ and $f_2$ is -0.867, and the tensile strain $\varepsilon_t$ is 0.0021 with a coefficient of variation of 10% . Compute the expectation and variance of $N_f$based on the first-order

[9.0 Marks]

## Tables, Figures, and Equations

### Table Q1-1: Observation

| Observer A | | | Observer B | | |
|---|---|---|---|---|---|
| ° | ' | " | ° | ' | " |
| 90 | 45 | 10 | 90 | 45 | 05 |
| 90 | 44 | 50 | 90 | 45 | 00 |
| 90 | 44 | 55 | 90 | 44 | 45 |
| 90 | 45 | 00 | 90 | 45 | 50 |
| 90 | 44 | 50 | 90 | 45 | 00 |
| 90 | 45 | 10 | 90 | 44 | 55 |
| 90 | 45 | 00 | 90 | 45 | 15 |
| 90 | 45 | 20 | 90 | 45 | 44 |

### Table Q3-1 Speed Observation Data

| Speed Class | Observed Frequency |
|---|---|
| $34 \leq X < 36$ | 2 |
| $36 \leq X < 38$ | 4 |
| $38 \leq X < 40$ | 8 |
| $40 \leq X < 42$ | 10 |
| $42 \leq X < 44$ | 25 |
| $44 \leq X < 46$ | 35 |
| $46 \leq X < 48$ | 45 |
| $48 \leq X < 50$ | 99 |
| $50 \leq X < 52$ | 80 |
| $52 \leq X < 54$ | 45 |
| $54 \leq X < 56$ | 30 |
| $56 \leq X < 58$ | 238 |
| $58 \leq X < 60$ | 8 |
| $60 \leq X < 62$ | 4 |
| $62 \leq X < 64$ | 2 |
| $64 \leq X < 66$ | 1 |

## Table Q3-2 Standard Normal Probabilities

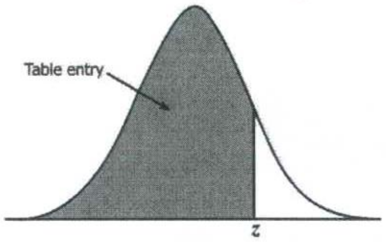


Table entry for $z$ is the area under the standard normal curve to the left of $z$.

| $z$ | .00 | .01 | .02 | .03 | .04 | .05 | .06 | .07 | .08 | .09 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0.0 | .5000 | .5040 | .5080 | .5120 | .5160 | .5199 | .5239 | .5279 | .5319 | .5359 |
| 0.1 | .5398 | .5438 | .5478 | .5517 | .5557 | .5596 | .5636 | .5675 | .5714 | .5753 |
| 0.2 | .5793 | .5832 | .5871 | .5910 | .5948 | .5987 | .6026 | .6064 | .6103 | .6141 |
| 0.3 | .6179 | .6217 | .6255 | .6293 | .6331 | .6368 | .6406 | .6443 | .6480 | .6517 |
| 0.4 | .6554 | .6591 | .6628 | .6664 | .6700 | .6736 | .6772 | .6808 | .6844 | .6879 |
| 0.5 | .6915 | .6950 | .6985 | .7019 | .7054 | .7088 | .7123 | .7157 | .7190 | .7224 |
| 0.6 | .7257 | .7291 | .7324 | .7357 | .7389 | .7422 | .7454 | .7486 | .7517 | .7549 |
| 0.7 | .7580 | .7611 | .7642 | .7673 | .7704 | .7734 | .7764 | .7794 | .7823 | .7852 |
| 0.8 | .7881 | .7910 | .7939 | .7967 | .7995 | .8023 | .8051 | .8078 | .8106 | .8133 |
| 0.9 | .8159 | .8186 | .8212 | .8238 | .8264 | .8289 | .8315 | .8340 | .8365 | .8389 |
| 1.0 | .8413 | .8438 | .8461 | .8485 | .8508 | .8531 | .8554 | .8577 | .8599 | .8621 |
| 1.1 | .8643 | .8665 | .8686 | .8708 | .8729 | .8749 | .8770 | .8790 | .8810 | .8830 |
| 1.2 | .8849 | .8869 | .8888 | .8907 | .8925 | .8944 | .8962 | .8980 | .8997 | .9015 |
| 1.3 | .9032 | .9049 | .9066 | .9082 | .9099 | .9115 | .9131 | .9147 | .9162 | .9177 |
| 1.4 | .9192 | .9207 | .9222 | .9236 | .9251 | .9265 | .9279 | .9292 | .9306 | .9319 |
| 1.5 | .9332 | .9345 | .9357 | .9370 | .9382 | .9394 | .9406 | .9418 | .9429 | .9441 |
| 1.6 | .9452 | .9463 | .9474 | .9484 | .9495 | .9505 | .9515 | .9525 | .9535 | .9545 |
| 1.7 | .9554 | .9564 | .9573 | .9582 | .9591 | .9599 | .9608 | .9616 | .9625 | .9633 |
| 1.8 | .9641 | .9649 | .9656 | .9664 | .9671 | .9678 | .9686 | .9693 | .9699 | .9706 |
| 1.9 | .9713 | .9719 | .9726 | .9732 | .9738 | .9744 | .9750 | .9756 | .9761 | .9767 |
| 2.0 | .9772 | .9778 | .9783 | .9788 | .9793 | .9798 | .9803 | .9808 | .9812 | .9817 |
| 2.1 | .9821 | .9826 | .9830 | .9834 | .9838 | .9842 | .9846 | .9850 | .9854 | .9857 |
| 2.2 | .9861 | .9864 | .9868 | .9871 | .9875 | .9878 | .9881 | .9884 | .9887 | .9890 |
| 2.3 | .9893 | .9896 | .9898 | .9901 | .9904 | .9906 | .9909 | .9911 | .9913 | .9916 |
| 2.4 | .9918 | .9920 | .9922 | .9925 | .9927 | .9929 | .9931 | .9932 | .9934 | .9936 |
| 2.5 | .9938 | .9940 | .9941 | .9943 | .9945 | .9946 | .9948 | .9949 | .9951 | .9952 |
| 2.6 | .9953 | .9955 | .9956 | .9957 | .9959 | .9960 | .9961 | .9962 | .9963 | .9964 |
| 2.7 | .9965 | .9966 | .9967 | .9968 | .9969 | .9970 | .9971 | .9972 | .9973 | .9974 |
| 2.8 | .9974 | .9975 | .9976 | .9977 | .9977 | .9978 | .9979 | .9979 | .9980 | .9981 |
| 2.9 | .9981 | .9982 | .9982 | .9983 | .9984 | .9984 | .9985 | .9985 | .9986 | .9986 |
| 3.0 | .9987 | .9987 | .9987 | .9988 | .9988 | .9989 | .9989 | .9989 | .9990 | .9990 |
| 3.1 | .9990 | .9991 | .9991 | .9991 | .9992 | .9992 | .9992 | .9992 | .9993 | .9993 |
| 3.2 | .9993 | .9993 | .9994 | .9994 | .9994 | .9994 | .9994 | .9995 | .9995 | .9995 |
| 3.3 | .9995 | .9995 | .9995 | .9996 | .9996 | .9996 | .9996 | .9996 | .9996 | .9997 |
| 3.4 | .9997 | .9997 | .9997 | .9997 | .9997 | .9997 | .9997 | .9997 | .9997 | .9998 |

## Useful Equations

$$\bar{X} = \frac{\sum_{i=1}^{K} n_i X_i}{N}$$

$$S = \sqrt{\frac{N \sum_{i=1}^{K} n_i {X_i}^2 - (\sum_{i=1}^{K} n_i X_i)^2}{N(N-1)}}$$

$$S_M = \frac{S}{\sqrt{N}}$$

$$\bar{X} - Z_{\alpha/2} S_M < \mu < \bar{X} + Z_{\alpha/2} S_M$$

$$E[g] = g(\mu) + \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} \left( \frac{\partial^2 g}{\partial x_i \partial x_j} \right) Cov[x_i, x_j]$$

$$V[x] = \sum_{i=1}^{n} \sum_{j=1}^{n} \left( \frac{\partial g}{\partial x_i} \right)_{\mu} \left( \frac{\partial g}{\partial x_j} \right)_{\mu} Cov[x_i, x_j]$$