UNIVERSITY OF RUHUNA

Faculty of Engineering

End semester examination in Engineering: July 2022

Module Number: CE3204   **Module Name: Structural Analysis I**

[Three Hours]

[Answer **all questions**, each question carries ten marks]

Q1. Straight elastic beam AB of uniform cross section with constant flexural rigidity of EI deployed to anchor boats in a jetty is shown in the Fig Q1. Beam is permanently fixed to the Jetty at point A and floats on top of a cylindrical buoy radius R (in m) at point B as shown in Fig. Q1. Beam is expected to support a uniform distributed load of intensity w kN/m at critical condition. Using three moment theorem, derive an expression for the reaction at the floating support B (symbolic form). Ignore the self-weight of the beam AB and the self-weight of the Buoy (float). Consider float is at same level of the fixed support when the beam is in unloaded conditions. The buoyancy force of the float (uplift force acting on the structure due to water pressure) for 1 m length of immersion shall be taken as k ($=\pi R^2 \rho g$). Assume that the water level remain constant along the section of the water way where the jetty is constructed.

[Three Moment Theorem equation with usual notations is as follows]

$$\frac{M_1 l_1}{E_1 I_1} + 2M_2\left(\frac{l_1}{E_1 I_1} + \frac{l_2}{E_2 I_2}\right) + \frac{M_3 l_2}{E_2 I_2} = 6\left[\left(\frac{\delta_1 - \delta_2}{l_1} + \frac{\delta_3 - \delta_2}{l_2}\right) - \left(\frac{A_1 a_1}{E_1 I_1 l_1} + \frac{A_2 a_2}{E_2 I_2 l_2}\right)\right]$$

[6 Marks]

a) Based on the results of the above and assuming that the beam is subjected to 12 kN/m at full load, complete the bending moment and shear force diagram for the beam AB and calculate the deformation of the beam at B at full load. Take the radius of the float (buoy) is 1m, span length (L) of the beam is 4m, the stiffness EI of the beam AB is 5×10^5 kNm², ρ=1000 kg/m³ and g=9.81 ms⁻².

[2 Marks]

b) Discuss the structural implication of the jetty beam during fluctuation of water levels in the waterway under current structural arrangement.

[2 Marks]

Q2. Fig. Q2. Shows a portal frame ABCD subjected to both horizontal and vertical loadings with stiffness values as indicated in the Figure. Joint B of the portal frame ABCD is subjected to horizontal load of 25 kN while the beam BC is subjected to 12 kN/m uniformly distributed load.

a) Considering only the vertical loading on the frame ABCD, calculate the bending moment of the joints using moment distribution method.

[3 Marks]

b) Calculate the bending moments of the joints of the frame ABCD by considering only the horizontal loading.

(Hint: In the calculation of bending moment of the frame due to lateral (horizontal) loading, first, assume joint moment for the unknown horizontal load P and thereafter calculated and calibrate the joint moments for the applied load based on the calculated values of the assumed horizontal load P).

[5 Marks]

c) Using the principal of superposition, produce the bending moment and shear force diagram of the frame ABCD using the results obtained from parts Q2. (a) and Q2.(b).

[2 Marks]

Q3. Fig. Q3 shows idealized frame structure ABC. The beam BC has twice the stiffness of the column AB. The beam, BC is expected to carry uniformly distributed load of 12kN/m, whereas 25 kN horizontal load is applied at the joint B. The flexural stiffness (EI) of the column element AB is 2.5×10^4 kNm^2.

a) Calculate the rotation and horizontal sway at joint B using slope deflection relationship and force equilibrium.

[6 Marks]

b) Based on the results obtained from part (a), calculate the bending moment at joints and hence complete the bending moment and shear force diagram of the frame ABC.

[4 Marks]

Q4 Fig. Q4(i) shows a statically determinate roof truss ABCDE spanning 8m and supported at points A and D. Roof loads are applied as point load to the joint B and C. All members behave linear elastically with extension due to applied load given by $e = FL/AE$.

a) Calculate force in members AB, BC, CD, AE, BE, CE and DE using any of the methods considering the load applied at B and C and the assumed load P_o at E (Fig Q4(ii).

[3 Marks]

b) Calculate the downward deformation of the joint E due to the applied load at B and C. Take Young's modulus as 205 kN/mm^2 and cross sectional area of the members 10.5 cm^2.

(Hint: Use a point load P_o at E in addition to the applied loads at B and C in computing the strain energy of the system (see Fig. Q4. ii)). Displacement at E due to applied load at B and C is equal to partial differential of the strain Energy/complementary strain energy of the system with respect to P_o when P_o is equal to zero).

[7 Marks]

Q5 Fig. Q5(i) shows a statically indeterminate frame structure proposed as an extension to the existing structure. Point A is fixed to the ground and point D is fixed to the existing building. Cantilever tip (Point B) of the member BCD is expected to carry 12kN point load. It is required to calculate deflection of the point B, cantilever tip of the frame.

a) Considering suitable virtual force systems and writing virtual work expression, calculate the support reactions at joints A and D.

(Hint: Fig Q5. (ii) and (iii) are considered two options of virtual force systems to generate addition equation to solve redundant forces of the frame ABCD)

b) Now, by considering suitable additional virtual force system, calculate the deflection at the tip of the cantilever end B of the beam BCD. Take the stiffness of the section (EI) as 2.5×10^4 kNm².
(Hint: Use Fig Q5. (i) with unit load at point B without the concentrated load as virtual force system to calculate the deflection at point B)

[3 Marks]

c) Complete the bending moment and shear force diagrams of the frame ABCD.

[2 Marks]

Q6 a) Explain the differences in action diagram of any function, i.e. bending moment shear force, reaction, and the influence line and the advantages of the influence line over the action diagram.

[2 Marks]

b) Fig.Q6 (i) shows a propped cantilever beam with a hinge at point D in the middle of the beam BC. Derive an expression for the reaction at B and C and fixed end moment at C for a unit load traversing in the beam ABC.

[3 Marks]

c) Point E is located 1 m right to the support B. Based on the support reactions, plot influence line of bending moment and shear force at D.

[3 Marks]

d) Using the answer to question 6 (c), calculate the bending moment and shear force at point E for the axel loads of a vehicle position and magnitude given in Fig. Q6(ii)
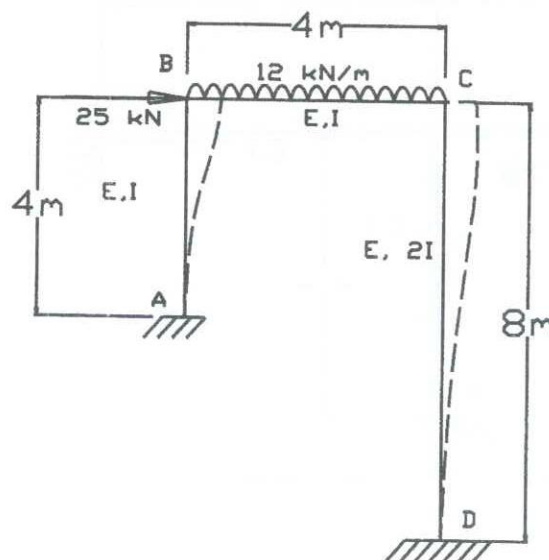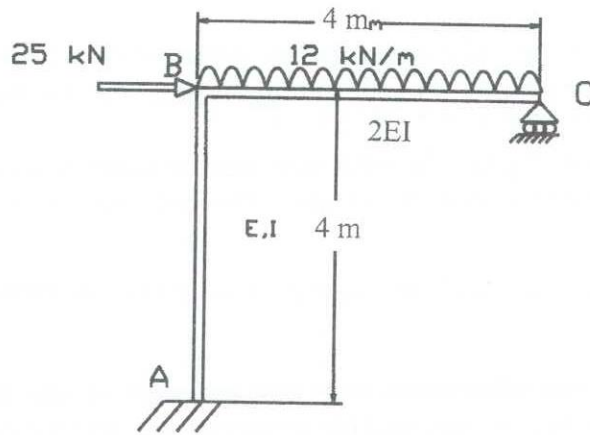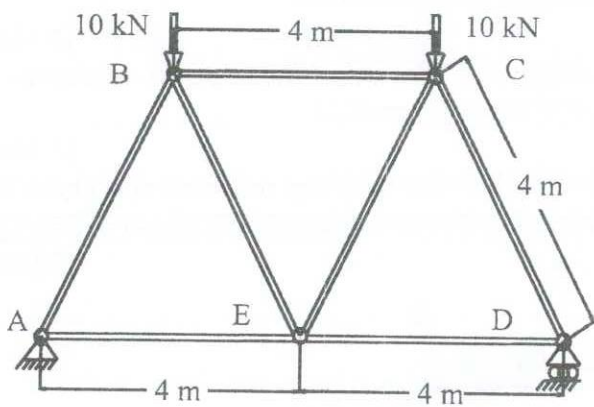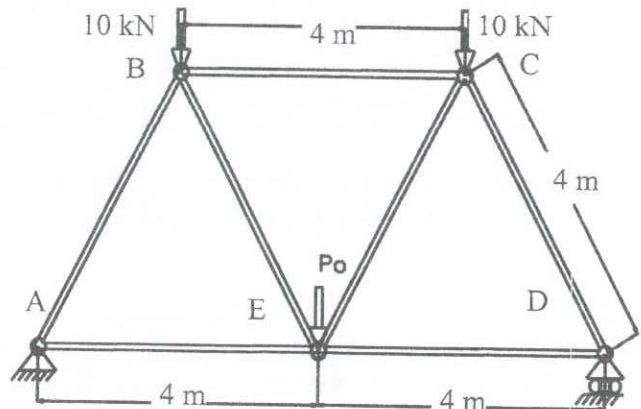
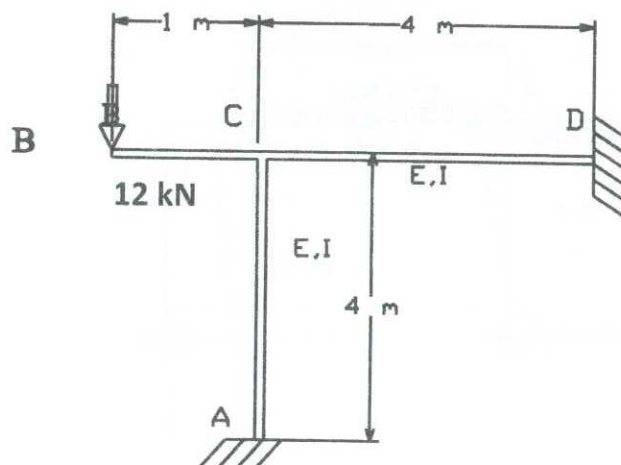[2 Marks]
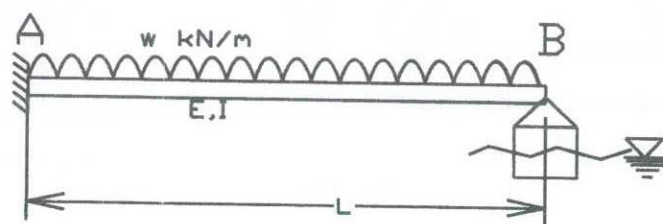
Fig. Q1

Fig. Q2
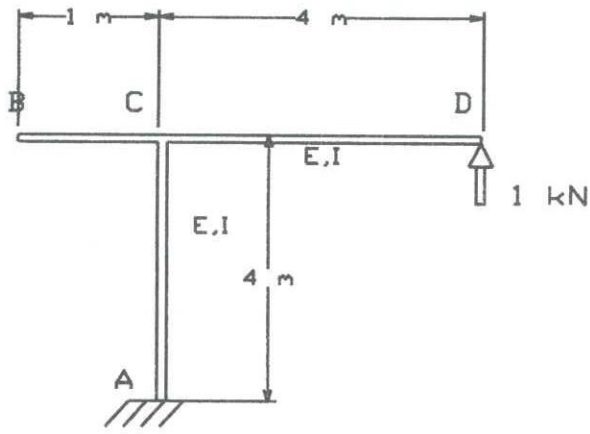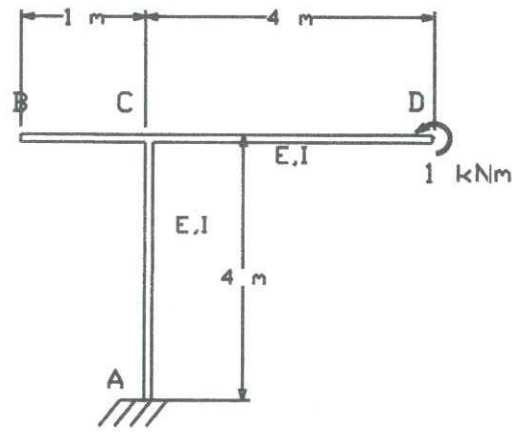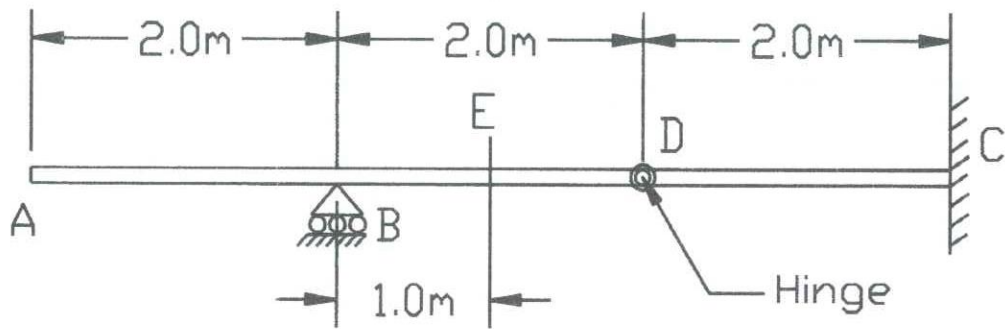
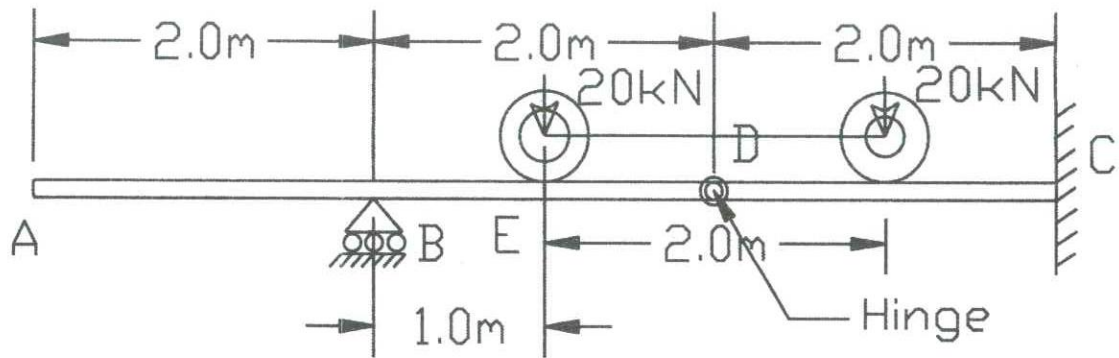Fig Q3

Fig. Q4(i)

Fig. Q4(ii)

Fig. Q4

Fig. Q5(i)

Q5(ii)

Q5(iii)

Fig Q5

Fig Q6(i)

Fig Q6(ii)

Fig Q6