University of Ruhuna- Faculty of Technology

Bachelor of Engineering Technology

Level 1 (Semester 2) Examination, December 2017

Course Unit: ENT1242 Electricity and Magnetism

Time Allowed 2 hours

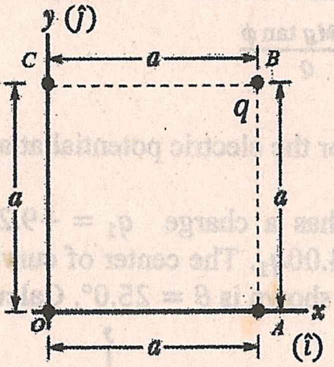
Answer all **Six (06)** questions

All symbols have their usual meaning

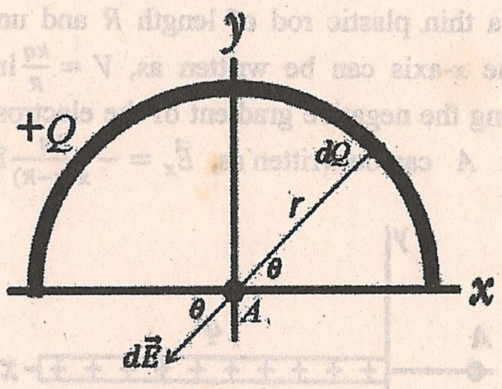
$[k = 8.99 \times 10^9\ \mathrm{Nm^2/C^2}, \varepsilon_0 = 8.85 \times 10^{-12}\ \mathrm{C^2/Nm^2}$, and $\mu_0 = 4\pi \times 10^{-7}\ \mathrm{T.m/A}]$

(1) (i) Write down a general expression for the electric field at a distance r from a point charge q.

(ii) In the following figure a particle of charge $q = 200$ C, is placed at one corner (B) of a square $OABC$ of side $a = 10.0$ cm as shown in the figure. Calculate the electric field at O and express your answer in unit vector notation.



(iii) A curved plastic rod of charge $+Q$, form a semi-circle of radius r in the xy plane as shown in the following figure. The x-axis passes through both end points and the charge is distributed uniformly on the rod. Starting from the equation $\vec{E} = \int_0^{\pi} d\vec{E} = -\frac{k}{r^2}\int_0^{\pi} \sin\theta \, dQ \ \hat{j}$, where $dQ = \frac{Q}{\pi} d\theta$, prove that the electric-field produced by the rod at point A can be written as, $\vec{E} = -\frac{2kQ}{\pi r^2}\hat{j}$.

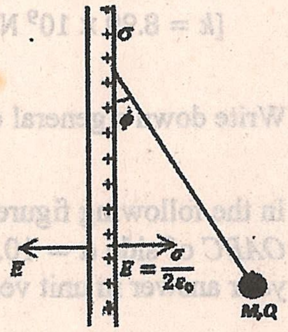


(2) (i) (a) Write down a general expression for the Gauss's Law.

(b) A straight, long wire has fixed positive charge with a linear charge density of magnitude λ. The wire is to be enclosed by a thin-walled, coaxial, non-conducting cylindrical shell of radius r. The shell is to have negative charge on its outside surface with a surface charge density σ that makes the net external electric field zero ($E = 0$) outside of the shell. Apply the Gauss's Law to a cylindrical Gaussian surface of length l and prove that $\sigma = -\frac{\lambda}{2\pi r}$. If $\lambda = 4.6\ \mu\text{C/m}$ and $r = 1.0$ cm then calculate σ.

(ii) (a) What is the electric force on a charge Q placed in an electric-field $\vec{E}$?

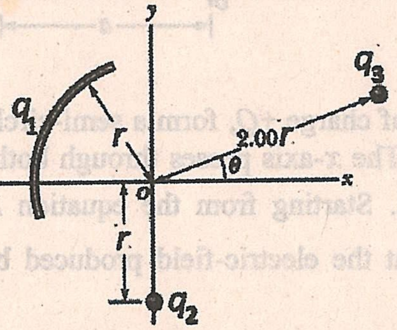
(b) A small, non-conducting ball of mass M and charge $+Q$ hangs from an insulating thread making an angle ϕ with a vertical, non-conducting sheet as shown in the figure. The sheet extends far vertically into and out of the page and it is uniformly charged. The charge of the ball is uniformly distributed throughout its volume and the electric field $E = \frac{\sigma}{2\varepsilon_0}$ due to the sheet is shown in the figure. The ball is in equilibrium position under the gravitational and electrical forces on it. Considering the forces, prove that the surface charge density of the sheet can be written as,



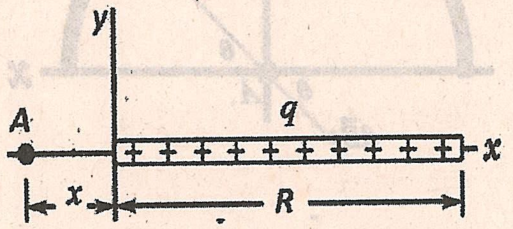
$$\sigma = \frac{2\varepsilon_0 Mg\tan\phi}{Q}.$$

(3) (i) Write down a general expression for the electric potential at a distance r from a point charge Q.

(ii) In the following figure, the arc has a charge $q_1 = +9.21\ \mu\text{C}$, and the two particles have charges $q_2 = 8.00q_1$ and $q_3 = -4.00q_1$. The center of curvature of the arc is at the origin and its radius is $r = 2.00$ m. The angle shown is $\theta = 25.0°$. Calculate the net electric potential at the origin (O) due to the three charges.



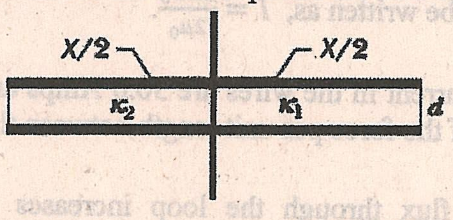
(iii) Following figure shows a thin plastic rod of length R and uniform charge q. The electric potential on point A on the x-axis can be written as, $V = \frac{kq}{R}\ln\left(1 - \frac{R}{x}\right)$ and the electric field can be found by calculating the negative gradient of the electrostatic potential (V). Prove that the electric-field at point A can be written as, $\vec{E}_x = -\frac{kq}{x(x-R)}\hat{\imath}$.



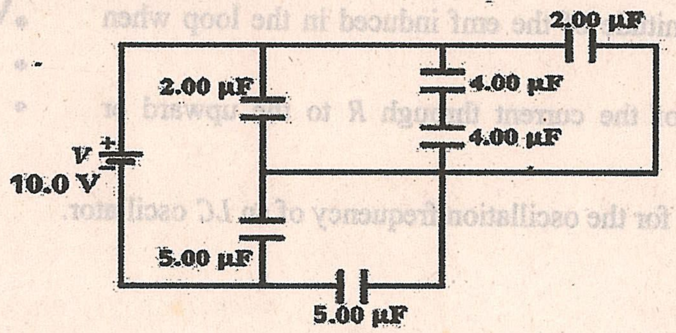
(4) (i) (a) The capacitance of a cylindrical capacitor is given by, $C = \frac{L}{2k\ln\left(\frac{r_b}{r_a}\right)}$. What are L, r_a, and r_b?

(b) A coaxial cable has an inner and outer radius of 0.20 mm and 1.20 mm respectively. Assuming that the cable is filled with air, calculate the capacitance per meter of the cable. [Hint: you can use the equation in part (a)]

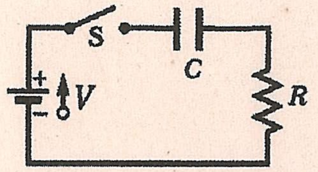
(ii) A parallel plate capacitor with a plate area X and separation d is shown in the following figure. The left and right half of the gap is filled with material of dielectric constant κ_2 and κ_1, respectively. Derive an expression for the total capacitance of the capacitor.



(iii) A 10.0 V battery is connected across six capacitors of capacitances that are labeled/shown next to them as shown in the following figure. What is the equivalent capacitance (C_{eq}) of the capacitor network connected to the battery?



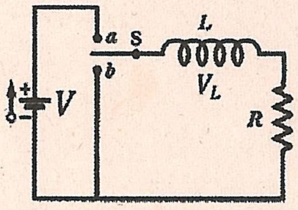
(5) (i) The switch S in the figure is closed at time $t = 0$, to charge an initially uncharged capacitor of capacitance C through a resistor of resistance R. The emf of the battery is V.



(a) If the charge of the capacitor at time t can be written as, $q(t) = Q_0\left(1 - e^{-\frac{t}{RC}}\right)$, then prove that the current through the capacitor at time t can be written as, $I(t) = \frac{V}{R}e^{-\frac{t}{RC}}$.

(b) Thus, if $C = 30.0\ \mu\text{F}$, $R = 10.0\ \Omega$, and the battery emf is $V = 10$ V, what is the potential difference across the resistor R at time $t = 0.21$ ms.

(ii) The switch S in the figure is closed on position a at time $t = 0$. The battery emf is V and the magnitude of the self-induced emf of the inductor-coil is V_L.



(a) If the current in the circuit at time t can be written as, $I(t) = \frac{V}{R}\left(1 - e^{-\frac{R}{L}t}\right)$, then using Kirchhoff's voltage/loop rule prove that the ratio of emf can be written as, $\frac{V}{V_L} = e^{\frac{R}{L}t}$.

(b) Hence, if $L = 1.10$ mH and $R = 100.0\ \Omega$, calculate the emf ratio $\frac{V}{V_L}$ at time $t = 7.62\ \mu\text{s}$.

(6) (i) Starting from the Ampere's Law, $\oint \vec{B}.d\vec{s} = \mu_0 I_{enc}$, prove that the magnetic field at a radial distance r produced outside by a long straight wire carrying a current I is given by $B = \frac{\mu_0 I}{2\pi r}$.

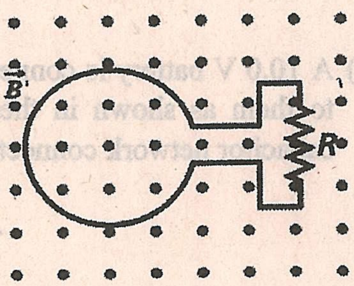
(ii) Consider that there are two long straight wires which are parallel and separated by a distance D. They are to carry equal currents such that at a point that is halfway between them they would produce a non-zero magnetic field B_0.

(a) Should the currents be in the opposite or the same direction in the two wires?

(b) Prove that the current can be written as, $I = \frac{\pi D B_0}{2\mu_0}$.

(iii) In the question above, if the current in the wires are 30.0 Amps and the separation $D = 8.0$ cm, then calculate the magnitude of the force per unit length between the two wires.

(iv) In the figure, the magnetic flux through the loop increases according to the relation $\Phi_B = 12.0t^2 + 14.0t$, where Φ_B is in webers and t is in seconds.



(a) What is the magnitude of the emf induced in the loop when $t = 2.0$ s?

(b) Is the direction of the current through R to the upward or downward?

(v) Write an expression for the oscillation frequency of an LC oscillator.