University of Ruhuna- Faculty of Technology

Bachelor of Engineering Technology
Level 1 (Semester 2) Examination, April 2019

Course Unit: ENT1242 Electricity and Magnetism
Time Allowed 2 hours

Answer all Five (05) questions

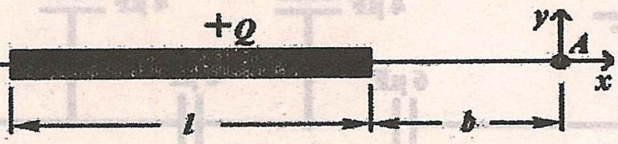
All symbols have their usual meaning
Briefer answers are anticipated whenever possible.
[$k_e = 8.99 \times 10^9\ \mathrm{Nm^2/C^2}$, $\varepsilon_0 = 8.85 \times 10^{-12}\ \mathrm{C^2/Nm^2}$, and $\mu_0 = 4\pi \times 10^{-7}\ \mathrm{T.m/A}$]

(1) (i) (a) State the Coulomb's law of electrostatic force.

(b) Write down the electric field at a distance r away from a point charge Q.

(c) What is the electrostatic force experienced by a point charge q placed in an electric field $\vec{E}$.

(ii) A non-conducting (i.e. insulating) rod of length $l = 7.15$ cm and charge $Q = 3.23$ nC distributed uniformly along its length is shown in the figure below.



(a) What is the linear charge density λ of the rod?

(b) Show that the electric field produced at point A, at a distance b can be written as,

$$\vec{E} = \frac{k_e \lambda l}{b(l+b)}\hat{\imath}.$$

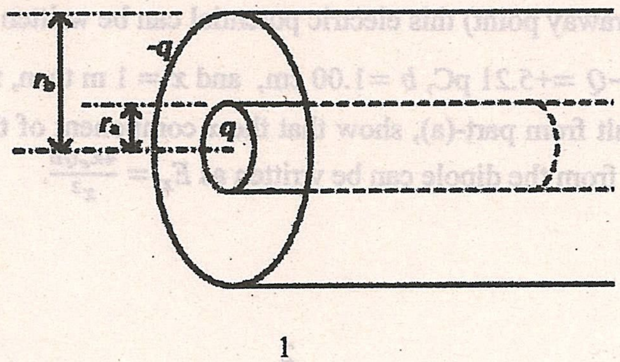
(c) Thus, find the magnitude and direction (relative to the positive direction of the x axis) of the electric field produced at point A, if $b = 10.0$ cm.

(d) Thus, find the magnitude of the electrostatic force on a particle of charge $q = 3.23$ nC placed at point A.

(e) If the point A is located faraway such that $b >> l$ then show that, $\vec{E} = \frac{k_e \lambda l}{b^2}\hat{\imath}$. [Hint: Use the result from part-(b).]

(2) (i) Write down the Gauss's law in electrostatics.

(ii) As shown in figure below a solid cylindrical conductor of radius r_a is surrounded by a coaxial conducting cylindrical shell of inner radius r_b. The length of both cylinders is L and you may neglect the edge effects (Assuming $r_b - r_a << L$). The inner cylinder has a charge $+q$ while the outer shell has a charge $-q$.



(a) Show that the electric field in-between the cylindrical conductor and the shell at a radius r (i.e. $r_a < r < r_b$) can be written as, $E = \frac{2k_e q}{rL}$.

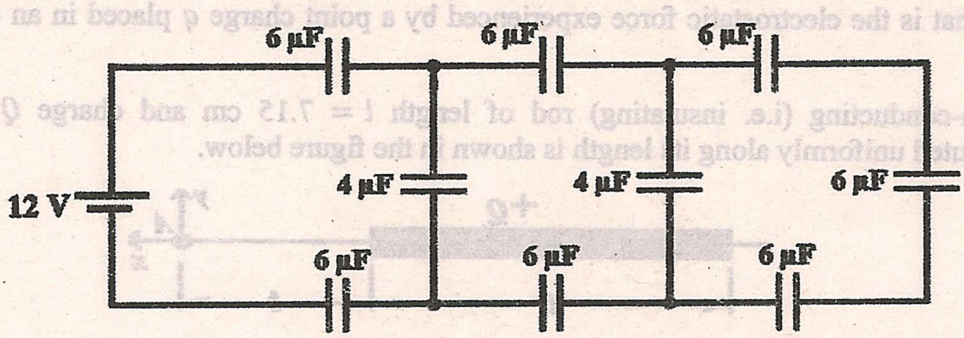
(b) Thus, show that the electric potential difference between the two conductors (i.e. cylindrical conductor and shell) can be written as, $\Delta V = -\frac{2k_e q}{L} \ln\left(\frac{r_b}{r_a}\right)$.

(c) Thus, if a dielectric material with dielectric constant κ is filled between the conductors, show that the capacitance of this cylindrical capacitor can be written as,

$$C = \frac{\kappa L}{2k_e \ln\left(\frac{r_b}{r_a}\right)}$$

(d) Assume that the space between the conductors is filled with a dielectric material having a dielectric constant of $\kappa = 3.0$. Calculate the capacitance of the capacitor, if $L = 10$ cm, $r_a = 0.20$ mm, and $r_b = 0.40$ mm.

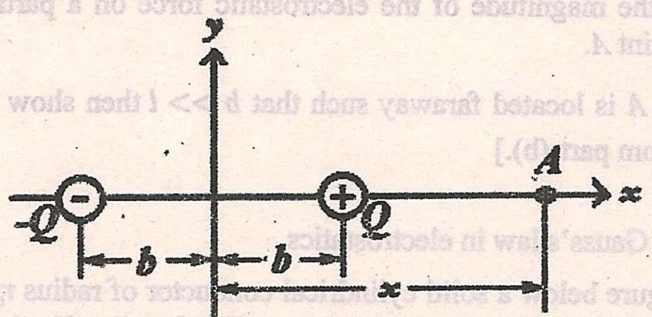
(iii) Find the equivalent capacitance to replace the network of capacitors connected across the battery that is shown in the figure below.



(3) (i) Write down the electric potential at a distance r from a point charge Q.

(Assume that the electric potential is zero at infinity)

(ii) Figure below shows an electric dipole.

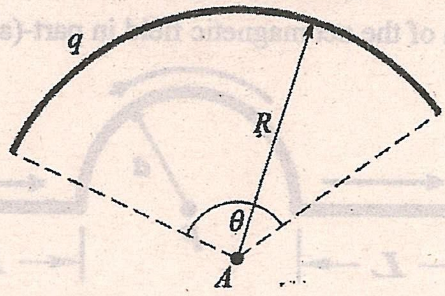


(a) Find the electric potential of the dipole at point A located at distance x, and show that when $x >> b$ (i.e. faraway point) this electric potential can be written as, $V_A = \frac{2k_e Qb}{x^2}$.

(b) Thus, if $Q = -Q = +5.21$ pC, $b = 1.00$ cm, and $x = 1$ m then, find V_A.

(c) Using the result from part-(a), show that the x component of the electric field at a faraway point ($x >> b$) from the dipole can be written as $E_x = \frac{4k_e Qb}{x^3}$.

(iii) The figure below shows a plastic rod that has been bent into a circular arc of radius R and central angle θ. The center of curvature of the rod is located at point A. If a charge q is uniformly distributed along the rod, show that the electric potential at point A can be written as $V = \frac{k_e q}{R}$. (You may assume the potential is zero at infinity)



(4) (i) (a) State the Kirchhoff's junction rule.

(b) State the Kirchhoff's loop rule.

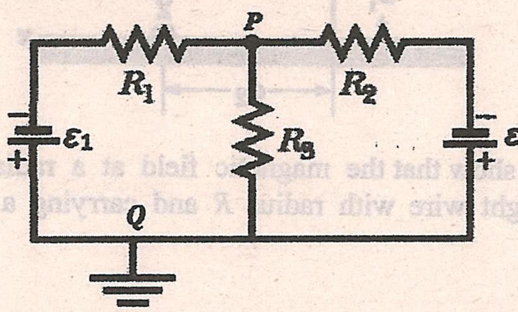
(ii) In the circuit shown in the figure below $R_1 = 30\ \Omega$, $R_2 = 20\ \Omega$, $R_3 = 10\ \Omega$, $\varepsilon_1 = 5.00$ V, and $\varepsilon_2 = 10.0$ V. Point Q of the circuit is grounded.

(a) Find the size and the direction (left or right) of the current through resistance R_1.

(b) Find the size and the direction of the current through resistance R_2.

(c) Find the size and the direction (up or down) of the current through resistance R_3.

(d) Find the electric potential at point P.

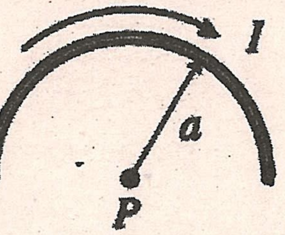


(iii) A V_0 potential difference is suddenly (at $t = 0$) applied across a resistor R and a capacitor C that are connected in series. At time t, the potential difference across the capacitor increases to V_c. [Note that the charge of the capacitor at time $t = t$ is given by $q = CV_0\left(1 - e^{-\frac{t}{\tau}}\right)$, where τ is the time constant.]

(a) Show that the current in the circuit at time $t = t$ can be written as, $i = \frac{V_0}{R} e^{-\frac{t}{\tau}}$

(b) Thus, show that the time constant of the circuit can be written as, $\tau = \frac{t}{\ln\left(\frac{V_0}{V_0 - V_c}\right)}$.

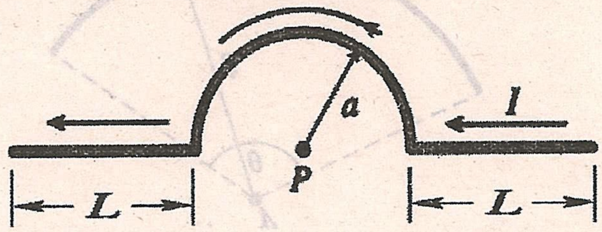
(5) (i) A semicircular wire arc of radius a and center P, carrying a current I is shown in the figure below. Using the Biot–Savart law show that the magnetic field produced at point P is given by, $B = \frac{\mu_0 I}{4a}$.



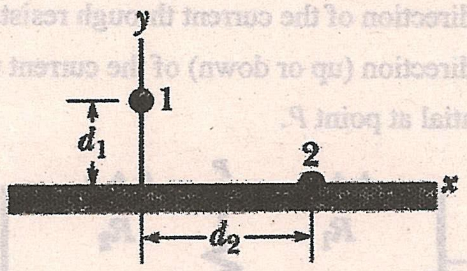
(ii) A wire is made of a semicircle of radius $a = 8.26$ cm and two straight sections (that are radial) each having length $L = 12.1$ cm, as shown in the figure below. The current in the wire is $I = 33.8$ mA.

(a) Find the magnitude of the net magnetic field at the center of curvature, P of the semicircle.

(b) State whether the direction of the net magnetic field in part-(a) into or out of the page.



(iii) The cross section of a long and straight wire 1 that carries a current of i_1 in to the page, is located at a distance d_1 from a surface as shown in the figure below. Another long wire 2, is located at a horizontal distance d_2 from wire 1 is parallel to the wire 1, and carries a current of i_2 out of the page. Show that the x component of the magnetic force exerted on a length L of wire 2 because of the wire 1 can be written as, $F_x = \frac{\mu_0 i_1 i_2 d_2 L}{2\pi(d_1^2+d_2^2)}$.



(iv) Using the Ampere's law, show that the magnetic field at a radial distance r produced inside (i.e. $r < R$) a long straight wire with radius R and carrying a current i can be written as, $B = \frac{\mu_0 i}{2\pi R^2} r$.