



# UNIVERSITY OF RUHUNA

## Faculty of Engineering

End-Semester 2 Examination in Engineering: February 2023

Module Number: CE2302

Module Name: Mechanics of Materials

[Three Hours]

[Answer all questions, each question carries 12 marks]

*All notations have their usual meanings*

- Q1. A simply supported beam of span 3 m carries a uniformly distributed load of intensity 48 kN/m. The cross section of the beam is a hollow box with wood flanges and steel side plates, as shown in Fig. Q1. The wood flanges are 100 mm x 75 mm in cross section, and the steel plates are 300 mm deep. The modulus of elasticity for steel and wood are 200 GPa and 10 GPa, respectively. The self-weight of the beam is negligible.
- a) i) Transform the given composite section in to an equivalent wood section and sketch the section indicating all the dimensions. [2.0 Marks]
- ii) For the transformed section obtained in Part i), show that the second moment of area about the z axis is given by  $I_z = 28.125 (7 + 3.2t) \times 10^6 \text{ mm}^4$ . [2.0 Marks]
- iii) If the allowable bending stresses for the steel and wood are 120 MPa and 6 MPa, respectively, determine the minimum thickness t of the steel plates required. [4.0 Marks]
- b) If a glued material is used to fix the wood flanges to steel plates having thickness calculated in Part a) iii), determine the minimum required shear strength of the glue. [4.0 Marks]
- Q2. a) State at least three conditions required to satisfy in applying the simple bending formula. [2.0 Marks]
- b) A timber beam spans 4 m is simply supported about both y and z axes, and it requires to support a concentrated vertical load  $P = 100 \text{ kN}$  passing through the diagonal AB at its mid-span as shown in Fig. Q2(a) and Fig. Q2(b). The beam orientation is as indicated in Fig. Q2(b) to satisfy the required appearance of the structure.
- i) Determine the applied force components,  $P_y$  and  $P_z$  along the y and the z axes, respectively. [2.0 Marks]

ii) Determine the maximum bending moments  $M_y$  and  $M_z$  in the beam about the y and the z axes, respectively.

[3.0 Marks]

iii) Find the maximum compression and tensile stresses in the beam, and indicate the maximum stress points in a sketch of the cross section.

[5.0 Marks]

Q3. The stresses at a point in a two-dimensional structural body is shown in Fig.Q3(a) with known normal stresses and unknown shear stresses  $\tau_{xy}$  at two mutually perpendicular planes.

a) Determine the value of  $\tau_{xy}$  for which the maximum tensile stress is equal to 60 MPa.

[3.0 Marks]

b) Using a proper scale, plot the Mohr's circle for stress condition given in Fig. Q3(a) using the calculated shear stress value in Part a).

[5.0 Marks]

c) Determine the magnitude and direction of the principal stresses and show them on a properly oriented stress block.

[3.0 Marks]

d) If the stress block is rotated  $30^\circ$  clockwise as shown in Fig. Q3(b), determine the normal and shear stresses on the rotated stress block.

Q4. a) Briefly state how Macaulay's method of integration is better over the normal integration method in calculating the deflection of elastic beams.

[2.0 Marks]

b) A simply supported beam has its supports 8 m apart at A and B as shown in Fig. Q4. It carries a uniformly distributed load of 6 kN/m between A and B starting from 1 m and ending at 5 m from the support A. The end B of the beam has an overhang of 1 m and at the free end a concentrated load of 8 kN is applied.

i) In order to use Macaulay's method to determine the slope and the deflection, write an expression for the bending moment.

[4.0 Marks]

ii) Using Macaulay's method of integration, determine deflection of the free end and the maximum deflection between points A and B. Use  $E = 120 \text{ GPa}$  and  $I = 20 \times 10^6 \text{ mm}^4$ .

[6.0 Marks]

Q5. a) A compound shaft made in steel and brass is shown in Fig. Q5. (a). If the compound shaft is subjected to torque T at its end, show that the torque shared by the steel shaft,  $T_s$  is given by,  $\frac{T G_s J_s}{G_s J_s + G_B J_B}$  where G and J, have their usual meanings.

[2.0 Marks]

- b) A steel shaft of total length  $L = 4.0$  m is encased for one-half of its length by a brass sleeve that is securely bonded to the steel shaft. The outer diameters of the steel shaft and brass sleeve are 70 mm and 90 mm, respectively. Shear modulus  $G$  for the steel and brass are 80 GPa and 40 GPa, respectively.
- Determine the allowable torque  $T_1$  that can be applied to the ends of the shaft if the angle of twist between the shaft ends A and C is limited to  $8.0^\circ$ . [4.0 Marks]
  - Determine the allowable torque  $T_2$  if the allowable shear stress in the brass and steel are limited to 70 MPa and 110 MPa, respectively. [4.0 Marks]
  - Determine the maximum allowable torque  $T_{\max}$  if all above conditions to be satisfied. [1.0 Mark]

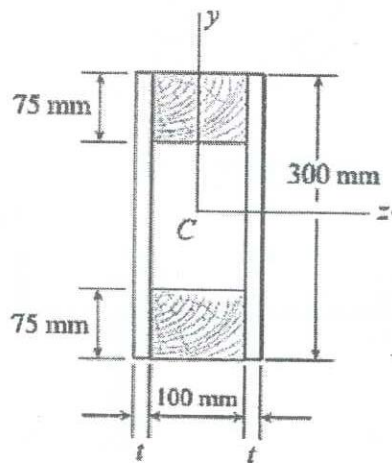


Fig. Q1

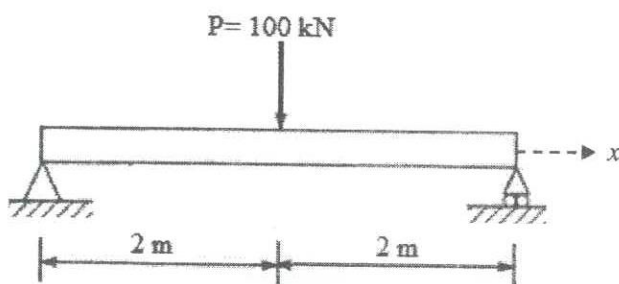


Fig. Q2(a)

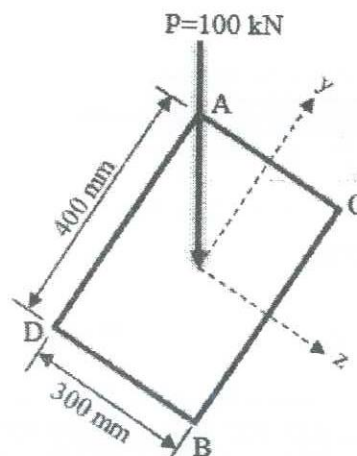


Fig. Q2(b)

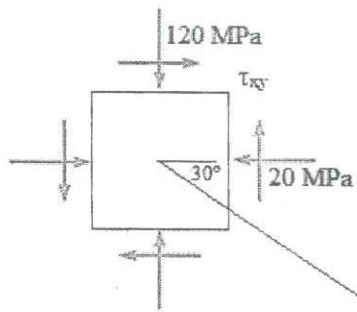


Fig. Q3(a)

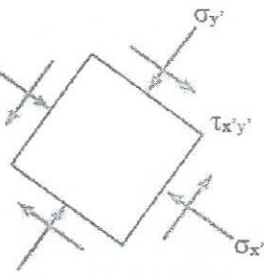


Fig. Q3(b)

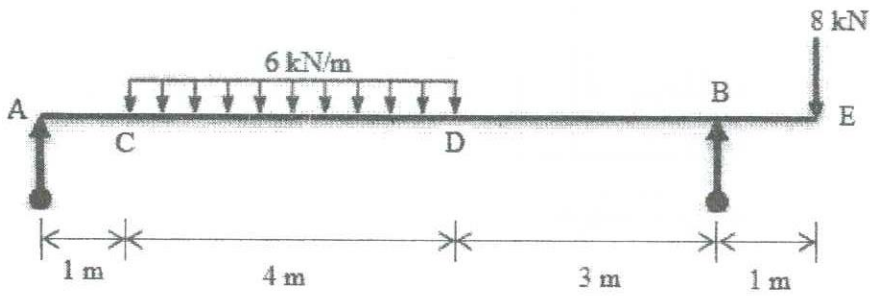


Fig. Q4

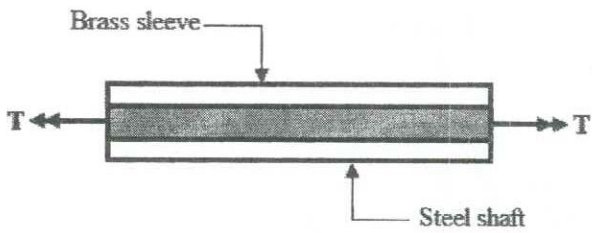


Fig. Q5(a)

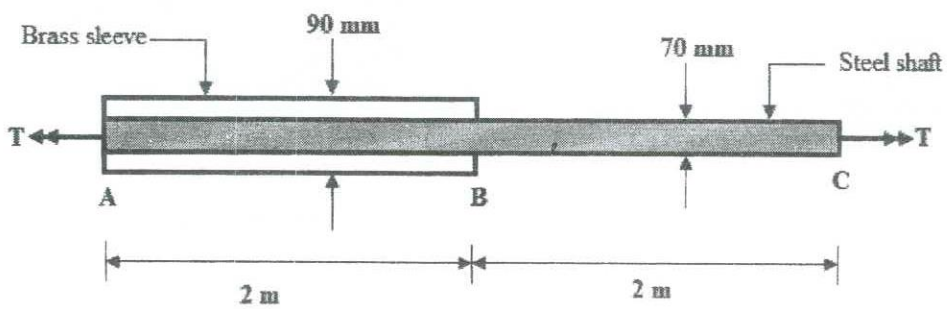


Fig. Q5(b)