University of Ruhuna- Faculty of Technology

Bachelor of Engineering Technology (honors) Degree

Level II (Semester II) Examination, September 2020.

Course Unit: TMS1213 Applied Calculus II

Time Allowed 3 Hours

All symbols have their usual meaning.

Note: Answer all six(06) problems.

1)

a) Briefly explain the Rolle's theorem.

b) In the given intervals of each functions below, find the points "c" that satisfy the conclusions of the Rolle's theorem.

i. $f(x) = x^2 - 4x + 3;\ [1,3]$

ii. $f(x) = \sin(x) + 2;\ [0,2\pi]$

c) Compute $\frac{dy}{dx}$ of following equations using implicit differentiation.

i. $x^3y^2 + 5xy + (xy)^2 = 1$

ii. $x^2 = \frac{x+y}{x-y}$

2) A solid is formed by revolving the curve $y = x^2 + 2$ around x-axis.

i. Graph the above function and sketch the shape of the solid object produced on the x interval [0,2].

ii. Using the method of slicing express the volume of the solid in terms of a summation of n number of slices.

iii. In the limit of number of slices (n) reaches to infinity (or else the thickness of the slice is negligibly small) the above sum will give the exact volume of the object. In this limit write the sum in definite integral form.

iv. Find the volume of the object by integrating.

3) Evaluate Following indefinite integrals. Use integration by parts if necessary.

a) $\int \sin(4x)\, dx$

b) $\int 2x\, e^{x^2} dx$

c) $\int x \sin(3x)\, dx$

d) $\int (x-1)\sqrt{5-x}\, dx$

Continued,

4)

a) Let $f(x) = -x^3 + 3x^2 - 4$

i. Find the intervals where the function is increasing and decreasing.
ii. Find the intervals where the function is concave up and concave down.
iii. Find the inflection points.

b) Consider the function $g(x) = 2x^3 + 3x^2 - 12x$ over the interval $-3 \le x \le 3$.

i. Find all the critical points.
ii. Find the absolute maximum and absolute minimum values.

5) Arc Length (L) of a curve $y = f(x)$ over the x interval $[a, b]$ can be given by,

$$L = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

Apply above formula to find the exact arc length of the following curves over the given interval.

a) $y = 2x^{\frac{3}{2}}$ from $x = 0$ to $x = 1$

b) $y = \frac{x^3}{6} + \frac{1}{2x}$ from $x = \frac{1}{2}$ to $x = 1$

6)

a) The surface area (S) of the surface of revolution that is generated by revolving a non-negative portion of a curve $x = g(y)$ from $y = c$ to $y = d$ about the $y-$axis can be expressed as

$$S = \int_c^d 2\pi g(y)\sqrt{1 + (g'(y))^2}\, dy$$

Use above formula to compute the area of the surface that is generated by revolving the portion of the curve $x = \sqrt{25 - y^2}$ from $y = -2$ to $y = 2$ about the $y-$axis.

b) Find solutions to the following differential equations subjected to the given initial conditions.

i. $\frac{dy}{dx} = 12x^2 - 8x + 3;\quad y(1) = 3$

ii. $\frac{dy}{dx} = \sin(3x) - \cos(2x) + 4e^{2x};\quad y(0) = 2$
