UNIVERSITY OF RUHUNA

Faculty of Engineering

Mid-Semester 3 (Old Curriculum) Examination in Engineering: June 2014

Module Number: ME 3305 **Module Name: Dynamics and Vibrations**

[Two Hours]

[Answer **all questions**, each question carries **five** marks]

A partial table of Laplace transformation is given in page 3. You may make additional assumptions where necessary, but clearly state them in your answers.

Q1. Let $f(t)$ be a function defined for all $t \geq 0$. Prove that the Laplace transform of the derivative of the $f(t)$ in given by.

$$\mathcal{L}\left[\frac{d\,f(t)}{dt}\right] = s\,\mathcal{L}\,[f(t)] - f(0)$$

Using this result:

a) Deduce an expression for the Laplace transform of the n^{th} derivative of $f(t)$.

b) Prove the initial value theorem

[5 Marks]

Q2. Find the inverse Laplace transform of

a) $F(s) = \dfrac{S+8}{s^2+4s+5}$

[2 Marks]

b) The mathematical model of a linear system is given by

$$\ddot{x}(t) + 2\xi\dot{x}(t) + \omega^2 x(t) = ay(t)$$
$$\dot{y}(t) + \beta y(t) + a\dot{x}(t) = v(t)$$

The input is $v(t)$ and the output is $x(t)$. Construct a block diagram of the system and find the system transfer function.

[3Marks]

Q3. Consider the mechanical system shown in Figure (Q3).Note that F acts on m_1 in the direction of x_1.

a) Calculate the transfer functions $H_1(s) = X_1(s)/\ F(s)$ and $H_2(s) = X_2(s)/\ F(s)$ in terms of the given parameters

[3.0 Marks]

b) Now let $m_1 = 25$ kg, $m_2 = 1$ kg, $k_1 = 100$ N/m, $k_2 = 10^4$ N/m and $b_1 = b_2 = 1$ Ns/m. Plot the poles/zeroes for both transfer functions.

[2.0 Marks]

Q4. a) Suppose a sinusoidal input ($u(t)=U_0 \sin \omega t$) has been applied on a dynamic system with the transfer function $G(s)$. If the system output is $y(t)$ with usual notations, Prove that:

$$y(t)=U_0 \, \| G(j\omega) \| \, \sin(\omega t+\varphi)$$

Briefly explain the significance of the above result.

[2.0 Marks]

b) A two wheel trailer of a tractor travels at a speed of 60 km/h over a road whose surface is sinusoidal with wave length of 20 m and amplitude of 30 mm as shown in Figure Q4. The mass of the trailer is 500 kg. It is supported by spring of total stiffness 25 N/mm and fitted with shock-absorber giving a damping ratio of 0.5. Considering only the single degree of freedom in the vertical direction, construct an equivalent mass spring dashpot with ground oscillation in order to simulate the trailer vibration. Hence, find the resulting amplitude of the trailer vibration.

[3.0 Marks]

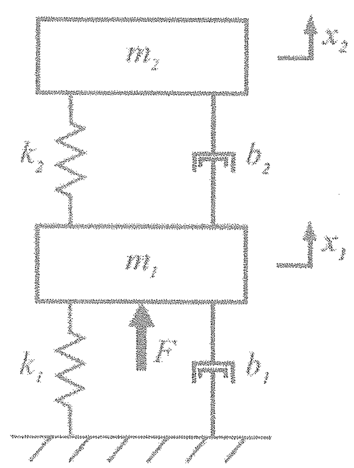

Figure (Q3)

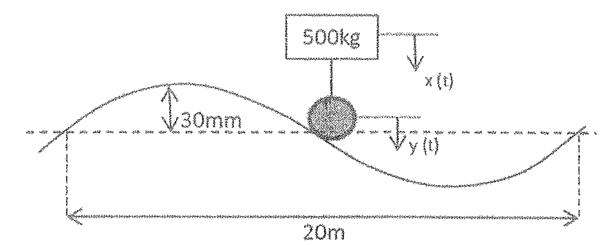

Figure (Q4)

Table of basic Laplace Transforms

$$\mathscr{L}\{f(t)\} = F(s) = \int_0^\infty e^{-st} f(t)\,dt$$

$f(t)$	$F(s)$		$f(t)$	$F(s)$
1	$\frac{1}{s}$		$u_c(t)$	$\frac{e^{-cs}}{s}$
t	$\frac{1}{s^2}$		$\delta(t)$	1
t^n	$\frac{n!}{s^{n+1}}$		$\delta(t-c)$	e^{-cs}
e^{at}	$\frac{1}{s-a}$		$f'(t)$	$sF(s)-f(0)$
$t^n e^{at}$	$\frac{n!}{(s-a)^{n+1}}$		$f''(t)$	$s^2F(s)-sf(0)-f'(0)$
$\cos bt$	$\frac{s}{s^2+b^2}$		$(-t)^n f(t)$	$F^{(n)}(s)$
$\sin bt$	$\frac{b}{s^2+b^2}$		$u_c(t)f(t-c)$	$e^{-cs}F(s)$
$e^{at}\cos bt$	$\frac{s-a}{(s-a)^2+b^2}$		$e^{ct}f(t)$	$F(s-c)$
$e^{at}\sin bt$	$\frac{b}{(s-a)^2+b^2}$		$\delta(t-c)f(t)$	$e^{-cs}f(c)$