University of Ruhuna- Faculty of Technology

Bachelor of Engineering Technology Honours

Level I (Semester I) Examination, June-July 2023

Academic Year 2021/2022

Course Unit: **TMS 1143 Physics of Mechanical Systems** Duration: **3 hours**

Instructions and details:

- Answer all **Six (06)** questions.
- Questions 1, 2 carry 16 marks each, and questions 1 to 4 carry 17 marks each.
- This question paper is composed of 4 pages.
- Calculators are allowed for calculations.
- When relevant, answers should be expressed in terms of the given (relevant) variables and simplified.
- You should neglect air resistance when solving problems.
- Strings/cords in problems have negligible mass and they do not stretch.
- All symbols have their usual meanings.
- $g = 9.81$ m/s^2.

1. Answer the following parts.

(i) At time $t = 0$ a projectile is launched in the xy plane from the origin with an initial speed v at an angle θ above the horizontal floor as shown in Figure-1. The y-axis is perpendicular to the floor and denote the acceleration of gravity by g. [*Note: Some common particle-motion equations: $v = v_0 + at$, $x - x_0 = v_0 t + \frac{1}{2} a t^2$, $v^2 = v_0^2 + 2a(x - x_0)$]

At time $t = t$ (see fig.), find the following functions of the projectile motion.

(a) $x(t)$. (i.e., position x as a function of time t)

(b) $y(t)$. (i.e., position y as a function of time t)

(c) $v_x(t)$. (i.e., speed v_x as a function of time t)

(d) $v_y(t)$. (i.e., speed v_y as a function of time t)

(e) $y(x)$. [i.e., position y as a function of position x. *Hint: Part-(a) and (b)]

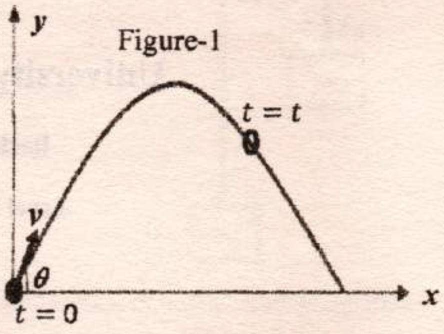


Figure-1

(ii) A point on a rim of a rotating disk gives its angular position as

$$\theta = -t^4 + 5.0t^2 - 4.0$$, where t is in seconds and θ is in radians.

(a) Calculate the angular velocity (ω) of the disk at $t = 3.0$ s. [*Note: $\frac{d}{dt}(t^n) = n\, t^{(n-1)}$.]

(b) Calculate the (instantaneous) angular acceleration (α) of the disk at $t = 5.0$ s.

2. Answer the following parts.

(i) As shown in Figure-2, two block masses m_1 and m_2 are connected by a string. m_1 is placed on a frictionless incline at an angle θ with the horizontal and the string is placed over a frictionless pulley such that m_2 hangs vertically. The mass of the pulley is negligible. The acceleration of gravity is denoted by g. If m_2 moves down, then

(a) Find the acceleration (a) of each block. (i.e., magnitude. *Hint: Newton's Laws)

(b) Find the tension (T) in the string.

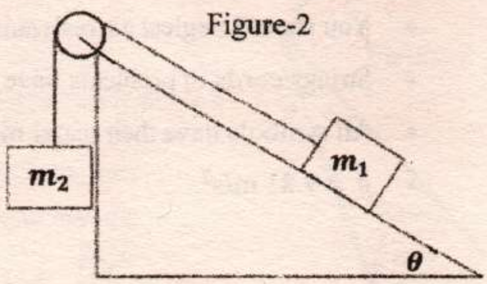
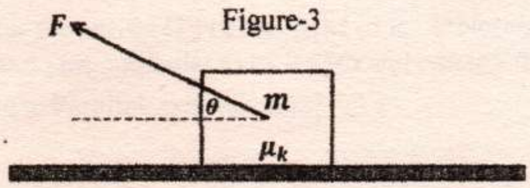


Figure-2

(ii) As shown in Figure-3, a force of magnitude F is applied on a block of mass m that is initially stationary on the floor. F makes an angle θ with the horizontal. The coefficient of kinetic friction between the block and the floor is μ_k. The acceleration of gravity is denoted by g. If the block moves horizontally without lifting then, find the acceleration (a) of the block. (i.e. magnitude)

Figure-3



3. As shown in Figure-4, a pendulum that is suspended from point C is pulled to the left side making an angle θ with the vertical and then released from rest. The pendulum is made of a cord of length L and a bob of mass m. As shown, a fixed peg A is located at a distance d from the point C making an angle α with the vertical. The acceleration of gravity is denoted by g. Then,

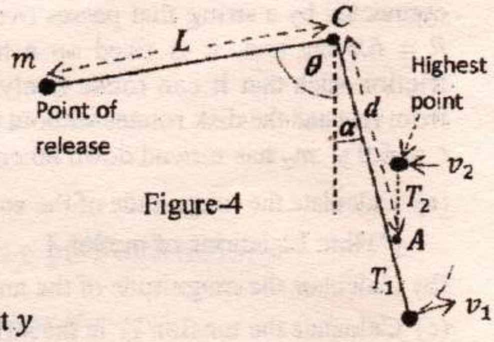


Figure-4

(a) For a mass m with a speed v that is located at a height y from a reference level, write expressions for the kinetic energy (K) and gravitational potential energy (U).

Immediately before the pendulum cord collides with the peg A,

(b) Find the speed v_1 of the bob. [See fig. *Hint: Mechanical energy conservation]

(c) Find the tension T_1 in the cord. [*Hint: Centripetal force]

After the pendulum cord collides with the peg A, the bob completes a full circle around peg A while the cord remains stretched. When the bob reached the highest point (see fig.),

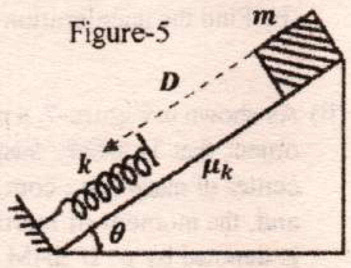
(d) Find the speed v_2 of the bob. [*Hint: You may first label as $r = L - d$]

(e) Find the tension T_2 in the cord.

(f) Find the minimum release angle θ, such that the bob merely completes the full circle. [*Hint: Part-(e)]

4. At the bottom of an incline of angle θ a spring with a spring constant k is fixed parallel to it as shown in Figure-5. From the top of the incline, a block of mass m is released from rest as indicated. Between the block and the incline, the coefficient of kinetic friction is μ_k. The block slides down the incline a distance D before it stops for a moment by compressing the spring by a length d. Neglect the mass of the spring. The acceleration of gravity is denoted by g. Then,

Figure-5



(a) When the block stops for a moment, what is the magnitude of the force exerted by the spring on the block?

(b) When the block stops for a moment, what is the elastic potential energy that is stored in the spring?

(c) Show that the increase of the thermal energy (of the system) can be written as

$$\Delta E_{th} = \mu_k mgD\cos\theta .$$

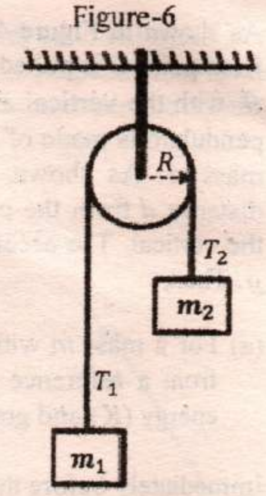
(d) State the principle of conservation of energy for an isolated system.

(e) Using the principle of conservation of energy, show that $D = \frac{kd^2}{2\,mg\,(\sin\theta - \mu_k\cos\theta)}$.

(f) Similarly, find the speed (v) of the block just as it touches the spring. [*Hint: You may first label as $X = D - d$]

5. As shown in Figure-6, two masses $m_1 = 450$ g and $m_2 = 550$ g are connected by a string that passes over a disk. The disk has a radius $R = 6.0$ cm and, it is fixed on a horizontal axle with negligible friction such that it can rotate freely. At time $t = 0$, m_1 is released from rest and the disk rotates without the string slipping on it. At time $t = 6.0$ s, m_2 has moved down 80 cm from its rest position. Then,

Figure-6



(a) Calculate the magnitude of the acceleration (a) of the blocks.
[*Hint: Equations of motion.]

(b) Calculate the magnitude of the angular acceleration (α) of the disk.

(c) Calculate the tension T_1 in the string. [*Hint: Newton's law/s]

(d) Calculate the tension T_2 in the string.

(e) Calculate the rotational inertia (I) of the disk.
[*Hint: Newton's law/s of Rotation].

(f) Calculate the mass (M) of the disk. [*Note: The moment of inertia of a disc about an axis perpendicular to it through the center of it is $I_{com} = \frac{1}{2}MR^2$]

(g) Calculate the angular speed (ω) of the disk at time $t = 6.0$ s. [*Hint: Rotational equations of motion.]

(h) Calculate the kinetic energy (K) of the disk at time $t = 6.0$ s.

6. Answer the following parts.

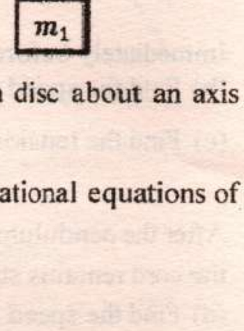
(i) A block-spring oscillator executes Simple Harmonic Motion (SHM). The position of the block at time t is given by $x(t) = x_m \cos(\omega t + \phi)$. Then,

(a) Find the velocity (v) of the block at time t. [i.e., magnitude. *Note: $\frac{d}{d\theta}(\cos\theta) = -\sin\theta$, and $\frac{d}{d\theta}(\sin\theta) = \cos\theta$]

(b) Find the acceleration (a) of the block at time t. (i.e., magnitude.)

(ii) As shown in Figure-7, a physical pendulum of mass m is made of a rigid object that is freely suspended from a frictionless pivot axis S. The center of mass (i.e., com) of the object is located at a distance h from S and, the moment of inertia of it about S is I. The acceleration of gravity is denoted by g. If SHM is been executed by the pendulum, then

Figure-7



(a) Starting from the Newton's law/s for rotation, show that the angular frequency of the SHM can be written as $\omega = \sqrt{\frac{mgh}{I}}$.
[*Hint/s: For SHM: $a = -\omega^2 x$. (i.e., part-(i) (b))]

(b) Find the period (T) of the SHM.

If the physical pendulum is a thin uniform rod of length l and mass m that is suspended freely from one end of it, then

(c) Find an expression for the period (T) of the SHM of it. [*Note/s: The moment of inertia of a thin rod about an axis perpendicular to its length and passing through its center of mass is $I_{com} = \frac{1}{12}ml^2$. *Hint/s: Parallel-axis theorem.]

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