UNIVERSITY OF RUHUNA

Faculty of Engineering

End-Semester 4 Examination in Engineering: January 2022

Module Number: IS4305 **Module Name: Probability and Statistics**

[Three Hours]

[Answer all questions, each question carries twelve marks]

Q1. a) Specimens of two different types of clay were selected and the percent shrinkage on drying was measured for each specimen, resulting in the following data.

Type I 16 29 16 15 24 20 24 21 17 11 18

Type II 12 23 19 24 17 20 18 19 11 18 8

i Construct a comparative box plot (a box plot for each sample with a common scale).

ii Comment on similarities and differences.

[6.0 Marks]

b) A subway system in city A has five inbounds and five outbound gates. The number of gates open in each direction is observed at a particular time of day. Assume that each outcome of the sample space is equally likely.

i What is the sample space?

ii Find the probability that at least two gates are open in each direction.

iii What is the probability that the number of gates is open in both directions is the same?

[6.0 Marks]

Q2. a) To find out the walking condition of a particular type of machine produced by a particular company, an investigation was carried out and observed a number of times a randomly selected 200 machines aged 3 and over from the date of production had shown errors. The investigation shows 17 machines with no errors, 30 machines with one error, 58 with two errors, 51 with three errors, 38 with four errors, and 7 with five errors. Assuming these proportions continue to hold exhaustively for the population of that production of the company,

i find the probability distribution of the number of times a machine aged 3 and over of this company has shown errors.

ii what is the expected number of times those machines had shown errors?

iii what is the standard deviation of the number of times those machines had shown errors?

[6.0 Marks]

b) Consider the point estimators for μ.

$$\widehat{\mu_1} = \frac{X_1}{3} + \frac{X_2}{3} + \frac{X_3}{3}$$
$$\widehat{\mu_2} = \frac{X_1}{2} + \frac{X_2}{3} + \frac{X_3}{6}$$
$$\widehat{\mu_3} = \frac{X_1}{2} + \frac{X_2}{3} + \frac{X_3}{3} + 2$$

Suppose that $E(X_1) = E(X_2) = E(X_3) = \mu$, $Var(X_1) = 7$, $Var(X_2) = 13$, and $Var(X_3) = 20$. Assume that the random variables X_1, X_2 and X_3 are independent.

i Calculate the bias of each point estimator. Is any one of them unbiased?

ii Calculate the variance of unbiased point estimators. Which point estimator has the smallest variance?

iii Obtain an expression to find the mean square error of a point estimator. Then use it to find the mean square error of $\widehat{\mu_2}$.

[6.0 Marks]

Q3. a) Component parts for an engine produced by company A are shipped to customers in lots of 100. If the specifications of the parts suggest that 95% of items meet the specifications, use a suitable approximation to find the probability that

i more than 3 items will be defective in a given lot.

ii less than 10 items will be defective in a lot.

[6.0 Marks]

b) Table 3.1 shows the number of errors found in 85 software products of a particular company.

Table 3.1

No. of Errors found in a soft-ware product	1	2	3	4	5	6
Frequency	17	20	25	14	6	3

Is there any evidence that the number of errors is modeled with a Poisson distribution with mean $\lambda = 3$? Use 5% level of significance.

[6.0 Marks]

Q4. a) The viscosity of two different brands of car oil is measured and their summary statistics are given in Table 4.1.

Table 4.1

Sample Statistics	Brand A	Brand B
Size	6	7
Mean	10.57	10.54
Variance	0.027	0.002

i Is the mean viscosity of the two brands is equal? Assume that the populations have normal distributions with equal variances. Use $\alpha = 0.05$.

ii Find the 95% confidence interval for the difference in mean viscosity.

[6.0 Marks]

b) An experiment was conducted to measure water pollution based on the quantity of dissolved oxygen. Four different locations were identified adjacent to a farm, a factory, a city, and a village and randomly selected water specimens. Table 4.2 displays the dissolved oxygen readings of the specimens from the four locations.

Table 4.2

Location	**Readings**
Location 1	6.5 5.4 5.0 4.8 6.0
Location 2	6.1 6.3 5.9 6.0 4.4
Location 3	5.1 6.4 6.3 5.0
Location 4	6.9 6.8 6.9 6.8 7.0

i Construct the one-way Analysis of Variance Table (ANOVA).

ii Do the data provide sufficient evidence to indicate the difference in mean dissolved oxygen content for the four locations? Use 5% level of significance.

[6.0 Marks]

Q5. a) An experiment was conducted to examine the effect of varying the water/cement ratio on the strength of concrete that had been aged 30 days and the obtained data presented in Table 5.1.

Table 5.1

Water/Cement Ratio	1.1	1.3	1.2	1.4	1.8	1.9	1.6	1.5	1.7	1.5
Strength	1.2	1.3	1.2	1.3	1.6	1.7	1.4	1.4	1.4	1.3

i Calculate the value of the correlation coefficient and interpret the result.

ii Consider the model; $\ln w = \ln \alpha + \gamma \ln k + \varepsilon$; where w and k are variables, $\ln \alpha$ and γ are regression coefficients.
By assuming the model fits data given in Table 5.1, find the regression coefficient to predict the strength of concrete of specified water/cement ratio.

iii Predict the strength of concrete when the water/cement ratio is at 2.0.

[6.0 Marks]

b) An experiment was carried out to assess the impact of the several variables force (gm), power (mW), temperature (°C), and time (msec) on ball bond shear strength (gm). By assuming that the ball bond shear strength is linearly related with all the other variables;

i State assumptions that use to construct multiple linear regression model to predict ball bond shear strength.

ii use matrix approach to explain the finding of regression coefficients and the regression equation for the given set of variables.

[6.0 Marks]