Parametric Modeling and Analysis of Tensegrity Structures

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Abstract

Tensegrity structures are based on a set of discontinuous compressible elements within a network of continuous tension elements, with isolated compressed elements (struts or bars) and prestressed tension elements (tendons or cables) that form a stable network. They are dominated by tensile elements, while more material-intensive compression elements are minimized. Tensegrity structures fail mainly due to low material efficiency, member instability, and excessive deflections when compared to rigid structures made with slender elements. The spatial geometry, axial stiffness, member layout, and connectivity of tensegrity structures directly affect the type of structural failure, including strength, instability, and stiffness. This paper presents a systematic parametric study on overall axial stiffness variation of the 3-bar tensegrity prism to check the effect of the level of prestressing and other geometric parameters such as the height of the tensegrity cell, type of the tensegrity cell (no. of compression members), radius of the tensegrity cell, area of the cables & struts, twisted angle of the top and bottom cable tringles and the point load acting on nodes. Parametric modeling and structural analyses were conducted in the Karamba 3D structural environment in-built with the Rhinoceros & Grasshopper 3D software.

***Keywords*—** Tensegrity networks, Prestress, Cables, Struts, Optimal configuration, Form-finding

Introduction

Background

Tensegrity structures are design principles based on a set of discontinuous compressible components within a network of components in continuous tension, with compressed elements (struts or bars) being isolated and prestressed tension elements (tendons or cables) that form a stable network [1]. They do not need to be anchored as they are "self-balanced and prestressed" [2], so they are independent of their weight or gravity. The word "tensegrity" is the combination of two words "tensional + integrity" [2] which refers to the structural integrity made up of tension and compression elements.

"Diamond", "Circuit" and "Zig-zag" configurations are the three main patterns of tensegrity cells based on their tendon patterns [3]. Any type of tensegrity structure can be obtained from combining these basic general modules.

These structures are lightweight compared to similar structures, have a high load-bearing capacity, sensitive to vibrations under dynamic loading, do not undergo buckling and torsion loads due to short struts, are cost economical, and can be extended endlessly [4]. But the main disadvantages are congestion among the bars when the structure becomes more complex and the lack of design and analyzing tools & techniques [5]. Furthermore, Tensile structures frequently deform when subjected to significant external forces, because the system is preferably only designed to support its own weight.

The advantages of Tensegrity structures that make this technology attractive for public use are the stability and financial feasibility of these materials. These designs make very effective use of the increased tensile strength characteristics that modern engineers have been able to extract from building materials [1]. Tensegrity structures are dominated by tensile elements, while more material-intensive compression elements are minimized. As a result, using tensegrity concepts to construct buildings, bridges, and other structures will make them both sustainable and cost-effective at the same time. Tensegrity structures have a broad variety of applications such as domes, roof structures, arches, furniture, space applications, robots, bridges, and towers, as well as computational modeling of human anatomy, macrocosm, and microcosm in biological sciences [6].

Related works

Tensegrity structures are made up of some general base modules (3-bar tensegrity, 4-bar tensegrity, etc. see Figure 1) [7]. When these base modules are linked, they form the final tensegrity design. Form-Finding entails creating an equilibrium matrix from nodal coordinates and topology [8]. There are usually two options in the Form-Finding phase of multi-module tensegrity structures.

The first is to consider a single module's equilibrium matrix and determine its possible self-stress vectors. The entire self-stress vector of the complete structure is then frequently assessed, considering the

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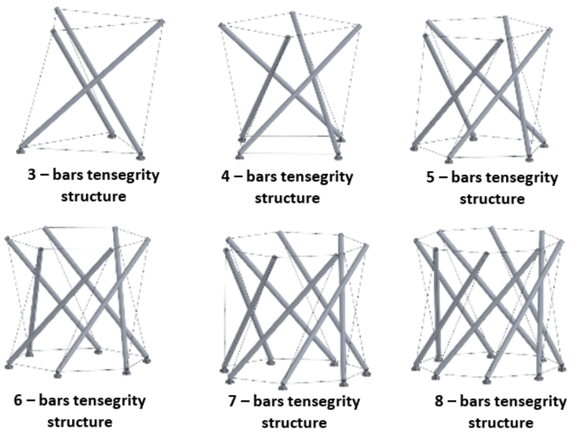


Figure 1: *Few types of tensegrity structures*

sharing components between modules. For example, by adding/superposing the values of these sharing elements or by scaling the fundamental module's self-stress vector and applying it to the remaining modules [9]. Figure 2(a) shows a single module tensegrity and according to the above-mentioned procedure by determining the self-stress vector for that module, the self-stress vector for the complete structure (Figure 2(b)) can be evaluated by superposing this single module.

The second approach is to consider the whole system's equilibrium matrix (all modules) at the same time (Figure 2(b)) and compute the self-stress vector of each connecting module (Figure 2(a)) using the method described above. In most of the literature, this selection has been widely revised as a basis of Form-Finding of tensegrity structures for analytical purposes [7].

In this research, form-finding methods are mainly focused on the design of tensegrity structures, which means the process of determining the best geometry

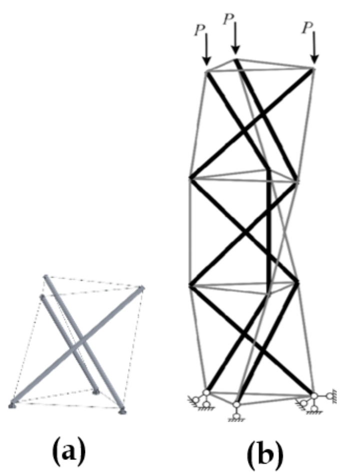


Figure 2: *(a) T-3 base module (b) Three-module T-3 tensegrity structure*

in the equilibrium stage [9]. Tensegrity structures are mostly failed due to the low material efficiency and high deflections when compared with rigid structures. Axial Stiffness of the tensegrity members directly affects the higher deflections in the structures, and it is varying according to the level of prestressing, area of the members, length of the members & other geometric parameters (horizontal and vertical declinations) [1].

Not adequate research has been conducted to check the axial stiffness variations of the tensegrity members. To achieve this, a parametric study is presented in this paper. This paper aims to parametrically simulate the general base module (T3 prism – 3 bar tensegrity) and study the influence of each parameter on the variation of axial stiffness of tensegrity structures to obtain an optimum structure. A general base module (T3-prism) is designed and analyzed using the structural analysis tool Karamba3D which was operating in the Grasshopper 3D that runs in the Rhinoceros 3D computer-aided software.

This paper is presented in the following order. Following the introduction, Section 2 describes the modeling of tensegrity structures. Section 3 elaborates on the parametric studies along with the modeling techniques. Section 4 elaborates on the results obtained from the parametric study and the reasons for the observations. Section 5 concludes the paper by highlighting the contributions of this work.

Modeling of tensegrity structures

The tensegrity structures are widely classified as prestressed and geodesic structures. Tensegrity prism, Diamond tensegrity, and Zig-zag tensegrity are the three primary classifications based on their tendon patterns. Diamond tensegrity is distinguished from Zig-zag tensegrity by the presence of struts surrounded by a diamond configuration of four tendons supported by two adjacent struts [2]. A 'Zig-zag' tensegrity is obtained from the diamond configuration as the basic structure with both ends of any strut should be connected by three non-aligned tendons arranged to form a 'Z' shape [3]. Tensegrity prisms are the simplest and one of the most instructive tensegrity structures in the tensegrity family. In this research paper, the simplest three-dimensional tensegrity unit, T3-prism was used to study the axial stiffness variation of tensegrity structures.

Mechanics of tensegrity structures

The tensegrity unit studied here is the T3- prism. It is also termed 3-bar tensegrity (Figure 3). The 3-bar tensegrity has nine cables and three struts which

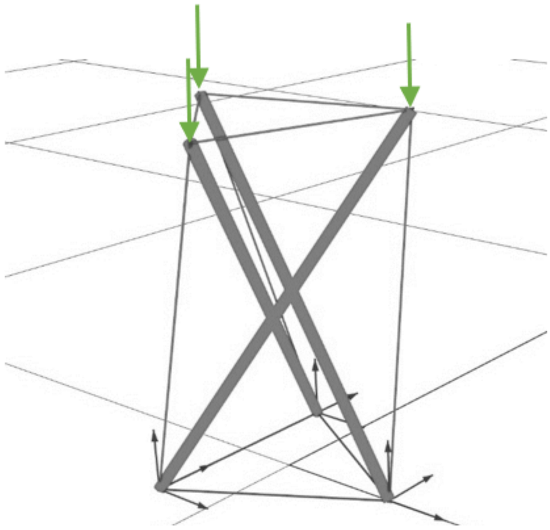


Figure 3: *3-bar tensegrity cell model*

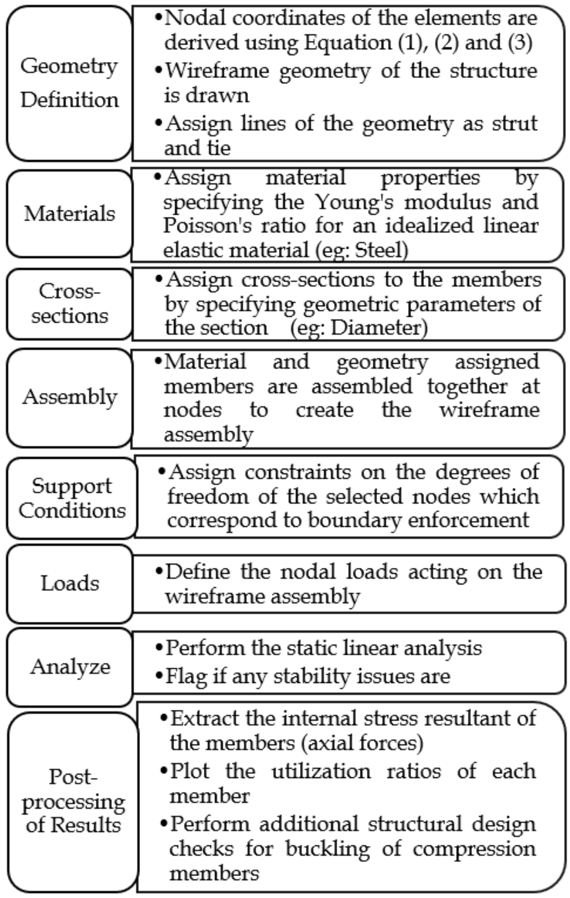


Figure 4: *Workflow of parametric modeling*

struts are isolated from each other. The top cables are connecting the top of each bar, the bottom cables connect the bottom of each bar, and the remaining cables connect the top and bottom of adjacent nodes. Figure 3 shows the 3D view of the 3-bar tensegrity cell. The two triangular faces on the top and bottom of this model are twisted with respect to each other. Otherwise, the structure will be unstable [2]. As it is a three-dimensional system, at each end of the strut we should have at least three cables attached to the node [5].

All members of tensegrity structures are either loaded in axial compression or tension [10]. This means the structure will only fail due to cable yield or buckling of struts. Since the compression members are not transmitting loads over a longer distance, they are not subject to higher buckling loads [3]. The prestressed cables are mostly used for these structures to have better stability to resist higher deflections. Only axial forces are present in these structures and neither bending moments nor shear forces are developed [1].

In this research, a 3-bar tensegrity cell was analyzed by assigning three-point loads in the z-direction (within the range of 0 – 5 kN) on the top nodes of the tensegrity cell to perform parametric modeling. The tensegrity cell was supported by assigning pin joints to all bottom nodes. The static analysis provides the minimum mass of the tensegrity structure by optimizing tensile forces in the cables and compressive forces in the struts in the presence of given external forces to solve for the minimum mass required under yielding constraints [11].

Parametric modeling of tensegrity cell

Karamba3D structural analysis tool was used to obtain the parametric results of the 3-bar tensegrity cell. It is fully embedded in the parametric design environment of Grasshopper, a plug-in for the Rhinoceros3D computer-aided software.

Figure 4 shows the workflow that follows to perform the parametric analysis using Grasshopper3D.

The axial stiffness of tensegrity structures varies according to several parameters like the area of members (cables & struts), length of members, twisted angle of the tensegrity structure, and level of prestress. Therefore, parametric analysis was carried out to design an optimum tensegrity structure with better stiffness, stability, and minimum mass.

Model pre-processing of 3-bar tensegrity cell

Length of triangular cable (l_t) was assumed as 30 cm and the height of the 3-bar tensegrity cell (h) was assumed as 50 cm. Then other properties were derived using the following equations. The radius of the 3-bar tensegrity cell (r), length of the strut (l_s), length of the bracing cable (l_c) are also defined.

$$r = \frac{l_t}{2\sin\left(\frac{\pi}{3}\right)} \tag{1}$$

$$l_s = \sqrt{h^2 + \left(2r^2 - 2r^2\cos\left(\frac{5\pi}{6}\right)\right)} \tag{2}$$

$$l_c = \sqrt{h^2 + \left(2r^2 - 2r^2\cos\left(-\frac{\pi}{6}\right)\right)} \tag{3}$$

The cables are assumed to have solid circular cross-sections whereas struts are assumed to be circular hollow to achieve an optimum structure [10]. The pretension of the cables was assumed to be zero for this analysis. To obtain the results of the variation in axial stiffness of the cell due to prestressed force, pretension was applied at a later stage. The above problem parameters constitute the control simulation for the parametric study.

Standard geometric parameters of the 3-bar tensegrity cell are presented in Table 1, and the material properties of the members (cables & struts) are presented in Table 2.

Table 1: *Standard geometric parameters of the 3-bar tensegrity cell*

Parameter	Value
Type of the tensegrity cell	3-bar
Twisted angle (degrees)	45
Compression hollow member outer diameter (cm)	1.6
Compression member thickness (cm)	0.3
Tension member diameter (cm)	0.3
Point Load (kN)	1

Table 2: *Material properties of the 3-bar tensegrity cell*

Properties	Value
Modulus of Elasticity (E)	210000 N/mm^2
Shear Modulus (G)	80760 N/mm^2
Density (ρ)	78.5 kg/m^3
Poisson's ratio (ν)	0.3
Steel Grade	235
Coefficient of Thermal Expansion (α)	1.00e-05 (1/°C)

Parametric studies

A parametric model is a computer-generated simulation of a design composed of geometric forms that have both constant and variable attributes. Variable characteristics are referred to as parameters. The designer can change the parameters in the parametric model to produce different alternative solutions to the problems in the model. However, employing traditional modeling methods to design more complex structures has proven difficult and time-consuming [12].

Parametric modeling is applied to simulate the axial stiffness variation of the tensegrity cell by changing the height of the tensegrity cell, type of the tensegrity cell (number of compression members), the radius of the tensegrity cell, area of the cables & struts, twisted angle of the top and bottom cable triangles, point load acting on the nodes, and the level of prestress.

Stability, displacement, and mass of the structure were taken as the results of the analysis to compare the stiffness variation of the tensegrity cell. Parametric analysis was performed by changing one parameter at a time while keeping the other parameters of the structure constant, as mentioned in Table 1. The stability of the tensegrity cell was represented by the utilization of compression and tension members (The ratio of the axial stress in the member to the yield stress of the material is referred to as utilization). Normally, as engineers, we practice achieving 85 - 95% of utilization to design the optimum structures. It derives that an extremely low percentage of utilization indicates a very stable structure (but not optimal financially) and a high percentage of utilization (>100%) indicates an unstable structure.

A stable equilibrium for the 3-bar tensegrity cell due to the variation of twisted angle was presented in Table 3. The geometry of the tensegrity cell will not be adequate to bear tensional loads in cables and compression loads in struts when the twisted angle is less than 30°. As well as when the twisted angle is more than 60° (according to the analyzed 3-bar tensegrity cell), the intersection of the compression members occurs in the middle of the tensegrity cell. Therefore, the twisted angle should be between 30°-60° to obtain a stable 3-bar tensegrity.

3-bar tensegrity refers to a tensegrity cell with three struts, whereas 4-bar tensegrity refers to a tensegrity cell with four struts. As the number of compression members rises, the tensegrity type will be altered. The number of compression members that can be provided for a given twisted angle influences the tensegrity structure's stability.

Table 4 shows the stability of the tensegrity cell

Table 3: *Stability due to twisted angle variation*

Twisted angle (θ)	Stability of tensegrity cell
$\theta \leq 30°$	Unstable
$30° < \theta < 60°$	Stable
$\theta \geq 60°$	Unstable

when its compression members are changed. The median value of 50° is taken because of the 3-bar tensegrity is stable between 30°- 60° range. The twisted angle should be in the higher range for sections to generate a stable tensegrity cell when increasing the struts. Therefore, 70° is selected when increasing the no. of struts in the tensegrity cell to obtain variation in the results.

Table 4: *Stability due to compression member variation*

Twisted angle (θ)	No. of Struts	Stability of tensegrity cell
50°	3	Stable
	4	Unstable
70°	3	Unstable
	4	Stable
	5	Unstable
	6	Highly Unstable

Results and discussion

It is restated that the buckling of compression members does not occur as the slenderness ratios of struts are very low. In the static analyses, due to the symmetry of the T3-prism, all compressive and tensile members have identical forces in them. Therefore, in this section, one such member from struts and ties is selected for the presentation of the results.

Figure 5(a) shows the variations of the mass, displacement, and utilization of tension & compression members against the height of the tensegrity cell. According to the results shown in the graph, as the height of the tensegrity cell increases, it becomes more stable due to the high utilization capacity of the members while also increasing its mass and displacements. As a result, the optimal structure height could be within the range of 20 - 45 cm.

Variations of the mass, displacement, and utilization of tension & compression members against the type of the tensegrity cell, the radius of the tensegrity cell, and point loads acting on the tensegrity cell

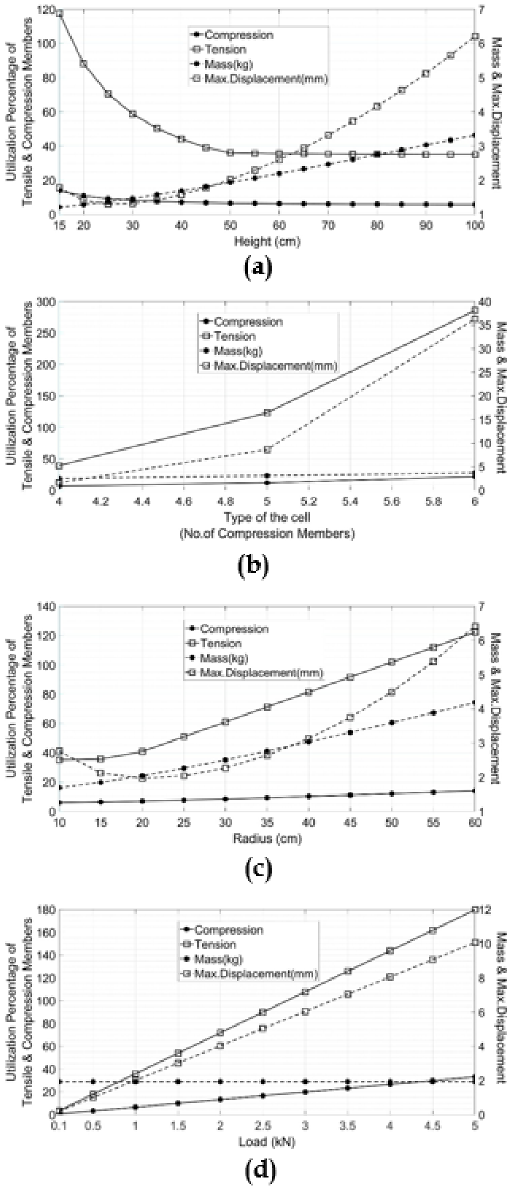
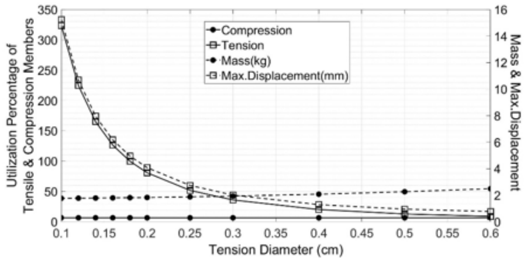
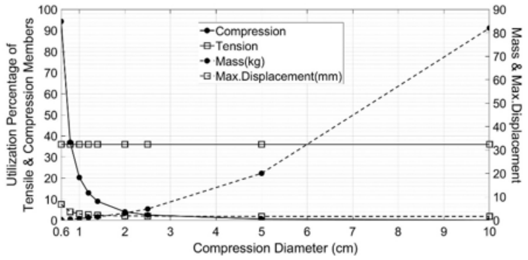


Figure 5: ***Variation of (a) height (b) type (c) radius (d) load on the structure***

are shown in Figure 5(b), Figure 5(c), Figure 5(d) respectively. Figure 5(b) is produced according to Table 4, taking the twisted angle as 70°. All three graphs show that the tensegrity cell tends to get more unstable when each parameter is increasing. The geometry of the tensegrity cell changes as the radius and number of struts increase, resulting in cell instability due to unfavorable stress distribution throughout the members. The stress acting on the members will increase as the load increases, resulting in structural instability. As a result, the optimal structure can be obtained by taking the radius within the range of 15 – 35 cm and the point loads on top nodes around 1.0 – 2.0 kN. Furthermore, it shows that the 4-bar tensegrity is the optimal structure rather than



(a)



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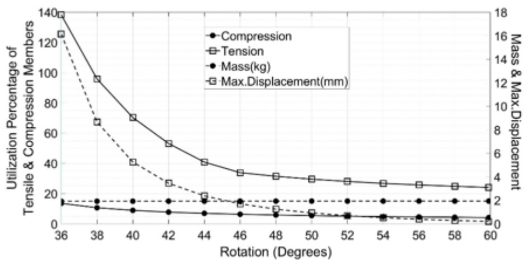

(c)

Figure 6: ***Variation of (a) tension member diameter (b) compression member diameter (c) twisted angle of the structure***

5-bar or 6-bar tensegrity cell.

Figure 6(a), Figure 6(b), Figure 6(c) depict the variations of the mass, displacement, and utilization of tension & compression members against the tension area, compression area, and twisted angle of the tensegrity cell. According to the results shown in the graphs, increasing those parameters causes the tensegrity cell to become more stable while simultaneously increasing its mass. As a result, both mass and stability should be considered while developing an optimal structure. The utilization capacity (in terms of the axial load-bearing capacity) of the members will increase as the cable and strut area increases. Increased twisted angle has a direct impact on the geometry of the tensegrity cell, resulting in cell stability due to improved stress distribution across the members. As a result, the optimal structure can be obtained by taking the compression diameter within the range of 0.6 – 1.0 cm and the tension diameter around 0.25 – 0.40 cm. Furthermore, it shows that the optimal tensegrity twisted angle could be within the range of 40° - 46°.

Figure 7 shows how the mass, displacement, and

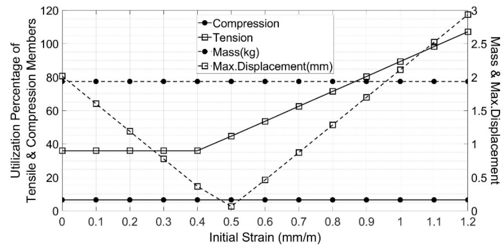

Figure 7: ***Variation against the Pretension***

utilization of tension and compression members change as the tensegrity cell is prestressed. Bottom cable tension is zero (without pretension), and top & braced cables have their maximum tensile stress due to point loads. When the initial strain is increased from 0 – 0.4 mm/m, the tensile force is transmitted to the bottom cable, which also reaches the structure's maximum tensile stress capacity. The structure's stability will remain constant up to this point. Further increase of strain only increases the tensile force in the bottom cables because other cables have already reached their maximum capacity, causing the structure to become unstable. As a result, the optimal structure strain could be within the range of 0.0 – 1.0 mm/m. In reality, all those parameters act on the performance of the 3-bar tensegrity cell simultaneously. Therefore, two optimum structures were designed and analyzed by taking each parameter within the given range to check whether the structure is truly optimal in order to validate the above-mentioned results which were taken by changing parameters individually. Structure A was derived by taking the parameters which give the best stable structure and structure B was derived by taking parameters around the median of the preferred permissible range.

Table 5 shows the selected parameters and the stability of the structure for both structures A and B. The structural utilization for both structures is less than 95%, hence it concludes that the structures are stable while taking all the parameters within the optimum range.

These kinds of 3-bar tensegrity cells can be combined to create a tensegrity tower. Additionally, due to a lack of expertise in this field, the actual applicability of these tensegrity concepts has not been thoroughly explored. However, these applications have the potential to handle a variety of loads, including earthquake, wind, and gravity, as well as sustain people inside of them. However, these principles are still developing their concepts in order to advance this technology. Instead of using conventional construction techniques, using tensegrity structures has various advantages as discussed in the paper. The stability of 3-bar tensegrity cells, which may be used to create tensegrity towers, is the focus of this re-

Table 5: *Optimal 3-bar tensegrity cells within the preferred range*

Parameter	Structure A	Structure B
Height (cm)	45	30
Radius (cm)	15	25
Load (kN)	1	1
Compression member outer diameter (cm)	1	0.75
Tension member diameter (cm)	0.4	0.3
Twisted angle (degrees)	46	43
Initial strain (mm/m)	0	0.5
Tensional Utilization	**18.30%**	**93.30%**
Compression Utilization	**19.20%**	**78.90%**

search study. Future research should concentrate more on understanding these general tensegrity configurations (T-prism, Diamond, and Zigzag), as they are used to create the most of tensegrity structures. An optimal tensegrity geometry can be defined by combining and scaling these fundamental tensegrity configurations after modeling and finalizing the solutions for tensegrity configurations.

Conclusions

This paper presented a parametric study to understand the axial strength, stiffness, and utilization variation of tensegrity structures. The structural analysis tool Karamba3D was used to model and analyze 3-bar tensegrity (T3-prism) in Grasshopper 3D that runs within a Rhinoceros 3D environment.

According to the results taken by analyzing the T3-prism, the optimal structure could be obtained by taking the radius within the range of 15 – 35 cm, the height within the range of 20 - 45 cm, the point loads on top nodes around 1.0 – 2.0 kN, the compression diameter within the range of 0.6 – 1.0 cm, tension diameter around 0.25 – 0.40 cm, the twisted angle within the range of 40° - 46° and finally, strain within the range of 0.0 – 1.0 mm/m.

The axial stiffness of tensegrity members has a direct effect on higher deflections in the structures, and it varies according to different parameters as mentioned above. Therefore, the above-presented parametric analyses were used to quantify the sensitivity of each parameter on the structural response of the tensegrity structures.

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