UNIVERSITY OF RUHUNA

Faculty of Engineering

End-Semester 5 Examination in Engineering: August 2014

Module Number: CE5217 **Module Name: Structural Analysis III**

[Three Hours]

[Answer all questions. Each question carries 12 marks]

All Standard Notations denote their regular meanings

Q1. a) What is the virtual work equation for yield line theory? Clearly define the parameters in the equation.

[2 marks]

b) Figure Q1(a) shows the isotropic slab is subjected to a vertical downward load of intensity '*p*' per unit area. Find required moment capacity of the slab.

[3 marks]

b) An isotropically reinforced rectangular slab shown in Figure Q1(b) is simply supported on all four edges. The slab carries a uniformly distributed load of 10 kN/m².

1. Draw possible yield line pattern at collapse.
2. Determine the collapse load corresponding to above yield line pattern.
3. Find the collapse load factor.

[7 marks]

Q2. A thin rectangular plate of x and y side dimension 'a', 'b' and thickness 't' is shown in Figure Q2. The plate is simply supported along all four edges. It is subjected to a vertical downward load of intensity, $p(x,y) = p_o \sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}$. The p_o is a constant and the m, n are positive integers.

a) Show that the trial solution for displacement $w = w_o \sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}$ satisfies the governing differential equation for thin plate when $w_o = \dfrac{p_o}{\pi^4 D\left(\frac{m^2}{a^2}+\frac{n^2}{b^2}\right)^2}$, here

$D = \dfrac{Et^3}{12(1-\nu^2)}$ is flexural rigidity of the plate.

[5 marks]

b) Show that the above solution satisfies the relevant displacement and stress boundary conditions.

[2 marks]

c) Determine the maximum deflection and its position, when m=2, n=2, a=b=1 m, t = 5 mm and p_o = 300 kPa. The Young's modulus (E) and the poison's ratio (v) of the plate material are 210 GPa and 0.3, respectively.

[5 marks]

Use the Governing differential equation and moment equation (with usual notation and sign convention) as shown below

$$\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^2 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = \frac{q}{D}$$

$$M_x = -D\left(\frac{\partial^2 w}{\partial x^2} + \nu\frac{\partial^2 w}{\partial y^2}\right) \text{ and } M_y = -D\left(\frac{\partial^2 w}{\partial y^2} + \nu\frac{\partial^2 w}{\partial x^2}\right)$$

Q3. A flat circular plate of radius b, is simply-supported concentrically by a tube of radius a, as shown in Figure Q3. If the internal portion of the plate is subjected to a uniform pressure p per unit area. Show that the central deflection δ of the plate is given by

$$\delta = \frac{pa^4}{64D}\left\{3 + 2\left(\frac{a}{b}\right)^2\left(\frac{1-\nu}{1+\nu}\right)\right\}$$

Use the circular plate differential equation and moment equation (with usual notation and sign convention) as shown below.

$$\frac{d}{dr}\frac{1}{r}\frac{d}{dr}\left(r\frac{dw}{dr}\right) = \frac{F}{D} \text{ and } M_r = D\left(\frac{d^2 w}{dr^2} + \frac{\nu}{r}\frac{dw}{dr}\right)$$

where $D = \dfrac{Et^3}{12(1-\nu^2)}$ and F is the shearing force / unit circumferential length.

[12 marks]

Q4. A spherical fuel oil tank of radius R sits on an annular ring support, as shown in Figure Q4. The oil density is ρ and the oil level is a distance bR from the crown of the tank. The self-weight of the tank can be neglected.

a) Drive the expressions for the meridianal stress in the sphere.

[10 marks]

b) Determine the meridianal stress just above the support ring and hence find the value of b for which this stress is numerically a maximum.

[2 marks]

Use the Membrane force equations for shell (with usual notation and sign convention) as shown below.

$$P_\varphi r_1 r - N_\theta r_1 \cos\varphi + \frac{\partial(rN_\varphi)}{\partial\varphi} = 0$$

$$P_r = \frac{N_\varphi}{r_1} + \frac{N_\theta}{r_2}$$

Q5. a) From the membrane force equations show that the membrane stresses in a conical shell (with usual notation and sign convention) are given by

$$N_\theta = P_r S \tan\alpha$$

$$N_S = \frac{1}{S}\int (P_r S \tan\alpha - P_S S)ds$$

[3 marks]

b) A conical shape shell is partially submerged in water of density ρ, as shown in Figure Q5. You can assume that the stresses due to self-weight of the shell are negligible. Determine the expressions for the membrane forces in conical part above and below the water level.

[9 marks]

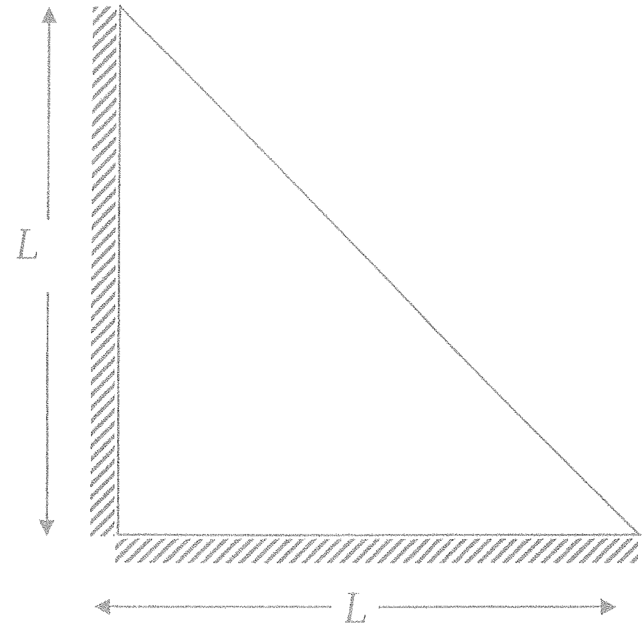

Figure Q1(a)

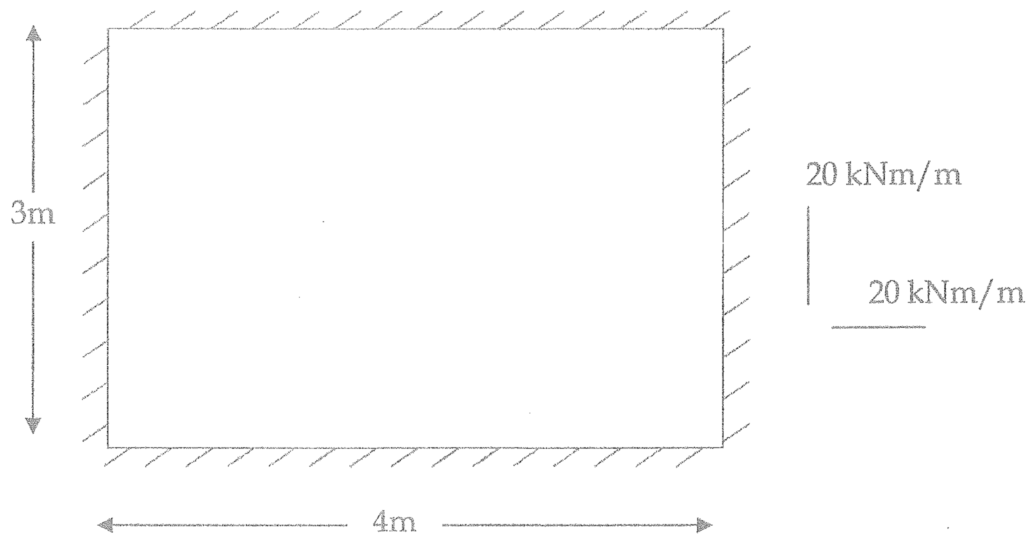

Figure Q1(b)

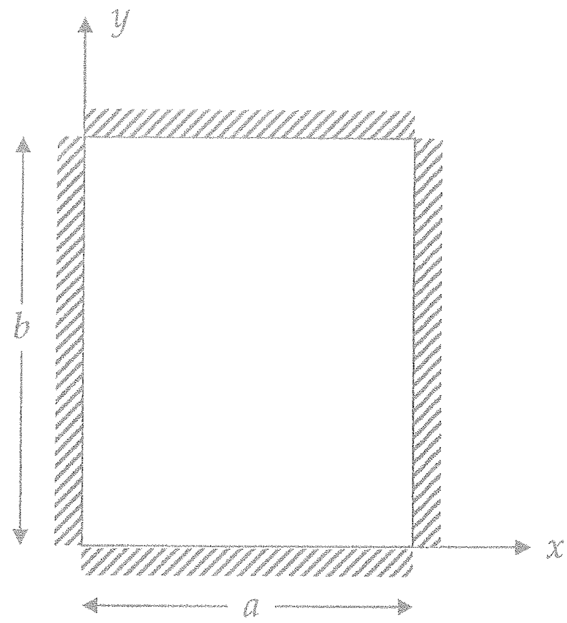

Figure Q2

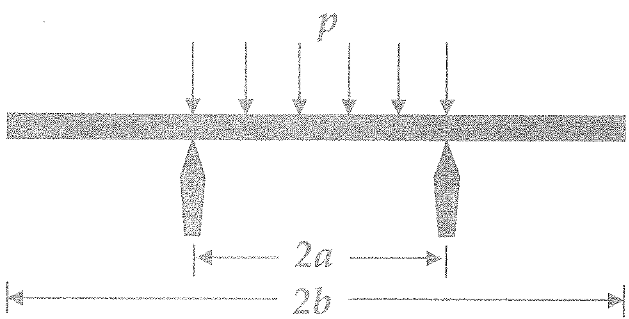

Figure Q3

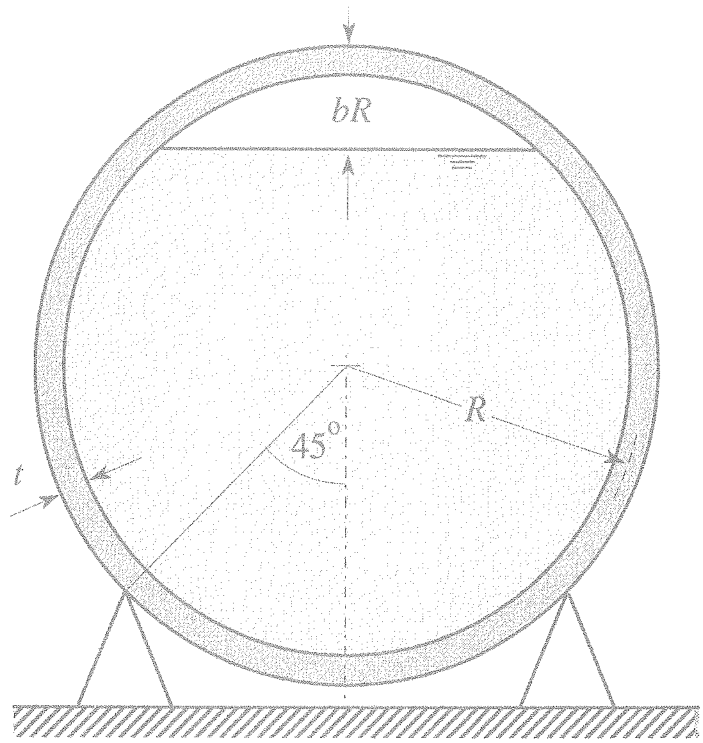

Figure Q4

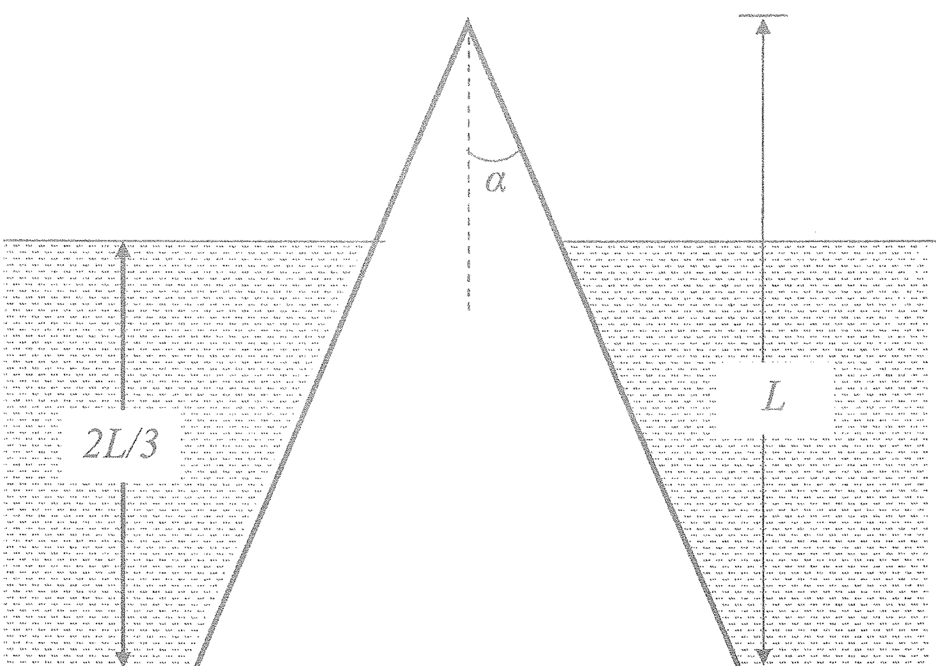

Figure Q5