UNIVERSITY OF RUHUNA

Faculty of Engineering

End-Semester 3 Examination in Engineering: February 2023

Module Number: CE3204 **Module Name: Structural Analysis I**

[Three Hours]

[Answer **all question**, each question carries ten marks]

Q1 a) Fig. Q1(a) shows beam with fixed ends subjected a uniformly distributed load W kN/m. Using moment curvature relationship prove that the fixed end moment at A and B; M_{FAB} and M_{FBA} is equal to $Wl^2/12$, where l is the length of the beam between support A and B

[4 Marks]

b) Floor level idealization of a beam in three-dimensional concrete frame along with its loading in the direction of the beam is shown in Fig. Q1 (b). Draw the bending moment diagram of the beam using principals of moment distribution. Assume that the beam stiffness changes from span to span as shown in the Fig Q1.

[6 Marks]

Q2. a) Express principals of virtual work on your own wards.

[2 Marks]

b) Propped cantilever beam shown in Fig. Q2 is loaded with a uniformly distribution load w kN/m and clockwise moment of W'L kNm applied at the center of the beam AB where W'=wL. Assuming a suitable virtual force system and writing expression for real deformation and calculate all the support reactions using principal of virtual work. Take the stiffness of the beam EI.
(Hint: Consider only the bending work of the system for the calculation)

[4 Marks]

c) Assuming a suitable virtual force system (virtual force system with unit downward force applied at the center of the beam AB), calculate the displacement of the center of the beam AB.

[4 Marks]

Q3. a) Express Castigliano's theorem of strain energy and complimentary strain energy in your own wards

[2 Marks]

b) Calculate the deflection at the tip of the cantilever shown in Fig Q3 considering both bending and shear energy. Consider Young's modulus of the material E, span of the cantilever L Shear Modulus (G=E/2). Take cross-section of the beam B (Breadth)x D (Depth).
(Hint. Consider point load Po at the tip of the cantilever for the determination of the deflection and evaluate the deformation due to bending and shear seperately)

[6 Marks]

c) Considering the answers above discuss span to depth ratio of the beam where the shear deformation become critical for the cantilever sections.

[2 Marks]

Q4 a) Portal frame structure ABCD with uniformly distributed load of 12kN/m applied on beam element BC is shown in Fig Q4. Using the slope deflection method calculate rotation of the joints B and C and thereby the moment at the joint B and C. Take the EI of the column members as 54×10^3 kNm^2 and EI of beam element as 108×10^3 kNm^2.

[4 Marks]

b) Using moment distribution method, recalculate the bending moments at joints B and C.

[3 Marks]

c) Compare the results obtained by the two methods and discuss the advantages and disadvantages of moment distribution method over slope deflection method in determining the bending moment diagram of the given structure

[3Marks]

Q5. a) State two theories involved in deriving the Three Moment Theorem. Using sketches to clarify your explanations derive the Three Moment Theorem (shown below) starting from the first principals.

$$\frac{M_1 l_1}{E_1 I_1} = 2M_2\left(\frac{l_1}{E_1 I_1}+\frac{l_2}{E_2 I_2}\right)+\frac{M_3 l_2}{E_2 I_2} = 6\left[\left(\frac{\delta_1-\delta_2}{l_1}+\frac{\delta_3-\delta_2}{l_2}\right)-\left(\frac{A_1 a_1}{E_1 I_1 l_1}+\frac{A_2 a_2}{E_2 I_2 l_2}\right)\right]$$

Note: All notation have their usual meanings

[4 Marks]

b) A straight elastic ground beam (tie beam) ABC of uniform cross section with constant flexural rigidity of EI (=256×10^3kNm^2) is subjected to point load transferred from the super structure applied at A,B and C equal to 1200kN, 1800kN and 1400kN respectively. Beam connected to soil encounter uniform subgrade reaction of 200,000 kN/m at A,B and C (Consider this as spring constant experience at ABC). The reactions at ABC are not necessarily equal to the load applied at ABC. Considering the reaction at A,B and C as R_a, R_b and R_c (unknowns), calculate the joint moment at B using three moment theory. Complete the bending moment diagram of the beam ABC and list reactions R_a, R_b and R_c.

[6 Marks]

Q6. a) Explain the differences between the action diagram and the influence line of any function (i.e. bending moment, shear force, reaction). Also explain the advantages of the influence line over the action diagram.

[2 Marks]

b) Fig.Q6 shows a continuous beam ABC with a hinge at point D, the middle of the beam BC. Derive expressions for support reactions at A,B and C when unit load move across the beam ABC and draw the influence line for the reactions at point A.B and C.

[4 Marks]

c) Point E is located 3 m right to the support A as shown in Fig. Q6. Based on the support reactions plot influence line of bending moment and shear force at E.

[4 Marks]

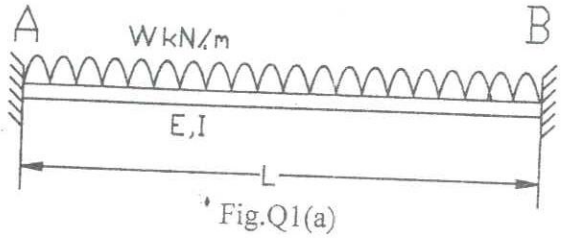

Fig.Q1(a)

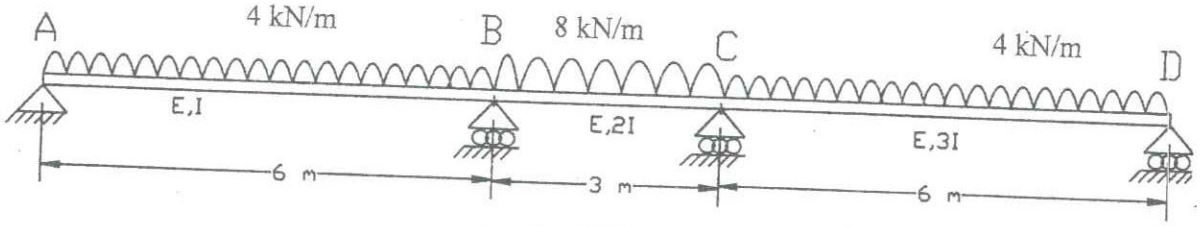

Fig.Q1(b)

Fig. Q1

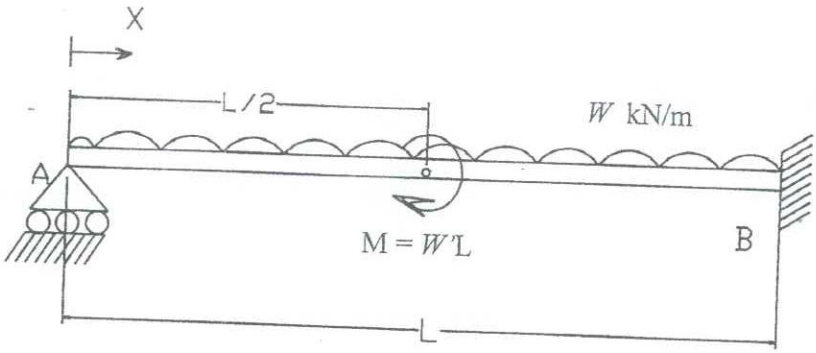

Fig. Q2

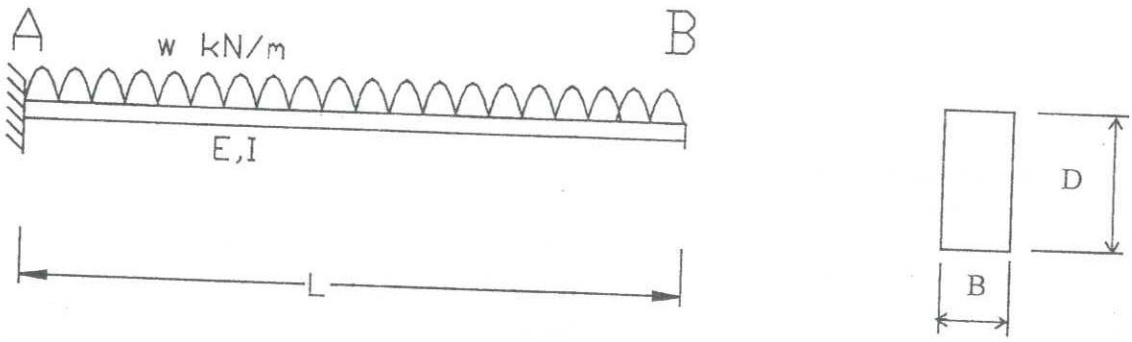

Fig. Q3

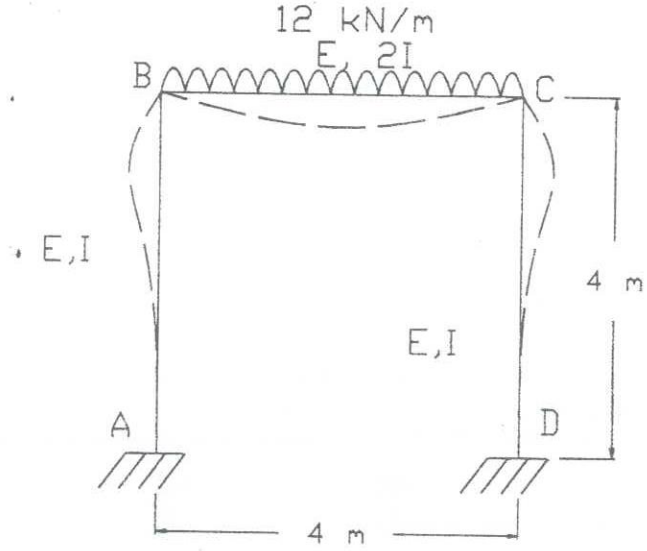

Fig. Q4

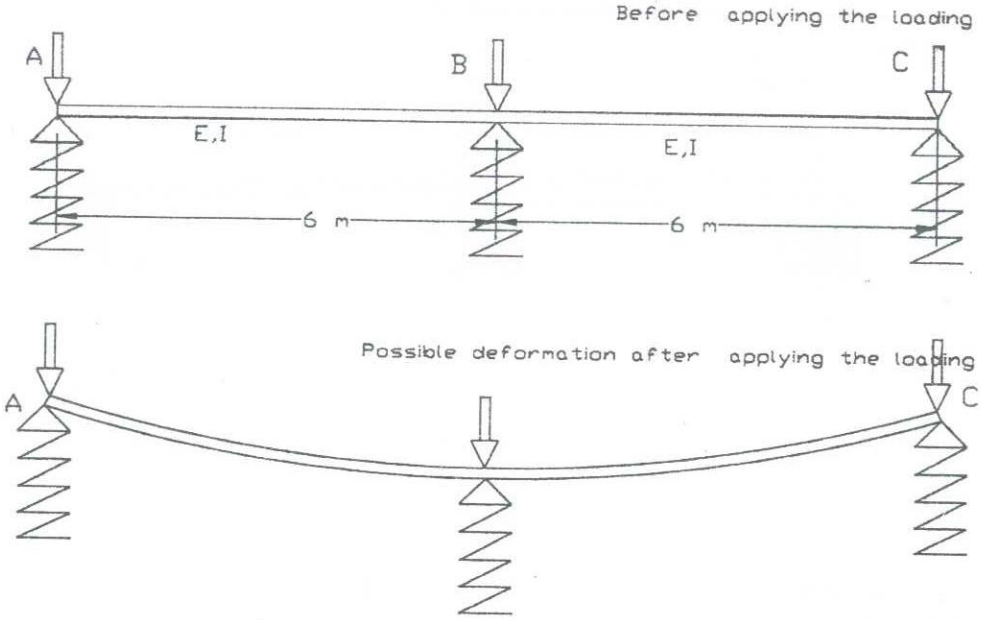

Fig. Q5

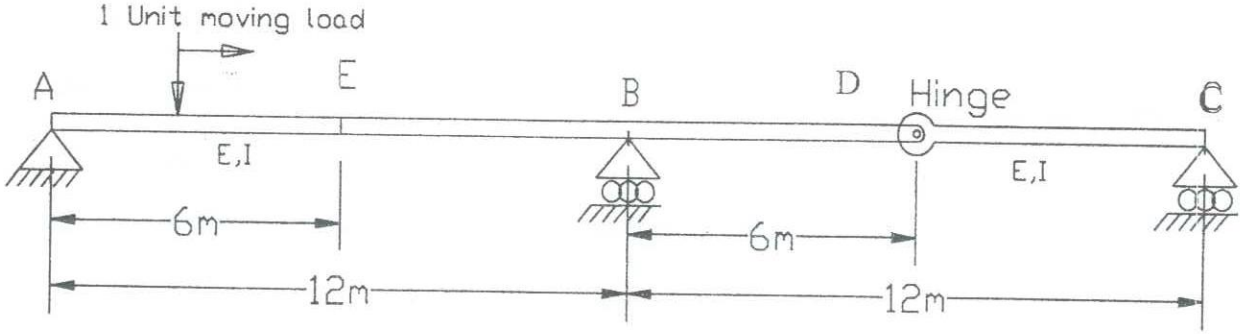

Fig Q6