# UNIVERSITY OF RUHUNA

## Faculty of Engineering

End-Semester 3 Examination in Engineering: February 2023

**Module Number: EE3305** **Module Name: Signals and Systems**

**[Three Hours]**

[Answer **all questions**, each question carries 10 marks]

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Q1 a) Consider the block diagram of an echo system shown in Fig. Q1. The input to the system is $f(t)$ and $\alpha_i, \tau_i > 0, i = 1, 2,$ are attenuation factors and delays respectively. The output $y(t)$ is the superposition of attenuated and delayed versions of the input. All the attenuation factors are less than unity.

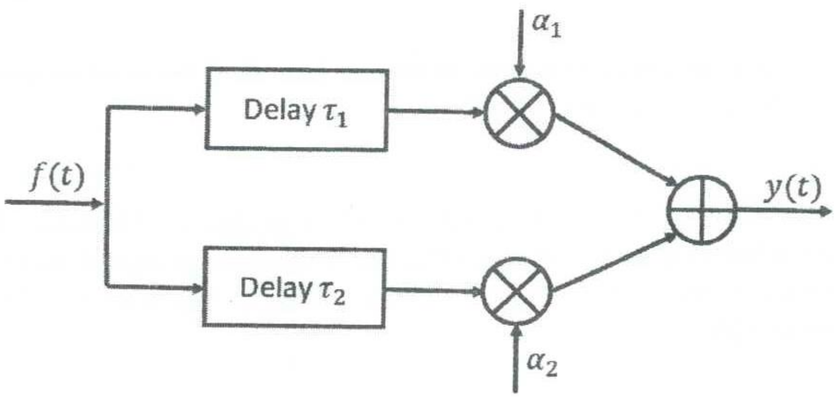


Fig. Q1

i) State a mathematical expression for the output, $y(t)$.

[1 Mark]

ii) State whether the given system is a causal, Bounded Input Bounded Output (BIBO) stable system. Justify your answer.

[2 Marks]

iii) Explain why a causal system does not need to be a memoryless system.

[1 Mark]

b) A discrete time Linear Time Invariance (LTI) system has the impulse response

$$h[n] = u[n+2] - u[n-2]$$

i) State a mathematical expression for the system response $y[n]$ when the input signal to the system is $f[n]$.

[2 Marks]

ii) Determine $y[n]$ when $f[n] = \alpha^n u[n]$ where $|\alpha| < 1$. Use the graphical convolution method to obtain the answer.

[4 Marks]

[**Hint:** Summation formula for geometric series is given by:

$$\sum_{k=n_1}^{n_2} \alpha^k = \frac{\alpha^{n_1} - \alpha^{n_2+1}}{1-\alpha}; |\alpha| < 1]$$

Q2 a) Fourier transform of a continuous time signal $f(t)$ is given by $F(\omega)$.

i) State a mathematical expression for $F(\omega)$.

[1 Mark]

ii) Show that the Fourier transform of the signal $f(t)e^{-j\omega_0 t}$ is

$$\mathcal{F}\left[f(t)e^{-j\omega_0 t}\right] = F(\omega - \omega_0)$$

[2 Marks]

iii) Use the result obtained in part a) ii) to find the Fourier transform of the signal $y(t) = f(t)cos(\omega_c t)$.

[2 Marks]

b) Consider the block diagram of an Amplitude Modulated Suppress Carrier modulation system shown in Fig. Q2. The message signal $m(t) = 2cos(200\pi t)$ and carrier signal $c(t) = 5cos(1000\pi t)$ multiply together to obtain the modulated signal $s(t)$.

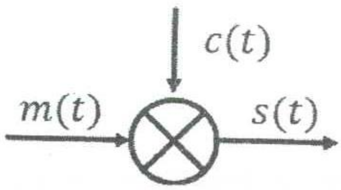


Fig. Q2

i) Find the Fourier transform of the modulated signal $s(t)$.

[1 Mark]

ii) Plot the magnitude spectrum of the modulated signal $s(t)$. Clearly indicate the frequencies and amplitudes.

[2 Marks]

iii) Assume an application requires the signal $z(t) = 4cos(800\pi t)$. Explain how an ideal low pass filter can be used to obtain $z(t)$ from the modulated signal $s(t)$. Clearly indicate the amplitude and cut off frequencies of the filter.

[2 Marks]

[**Hint:** Fourier transform of $e^{-j\omega_0 t}$ is given by $\mathcal{F}\left[e^{-j\omega_0 t}\right] = 2\pi\delta(\omega - \omega_0)$.]

Q3 a) State a mathematical expression for the Laplace transform of a continuous time signal $f(t)$.

[2 Marks]

b) Derive the Laplace transform for the unit step function.

[2 Marks]

c) Consider the circuit diagram shown in Figure Q3.a.

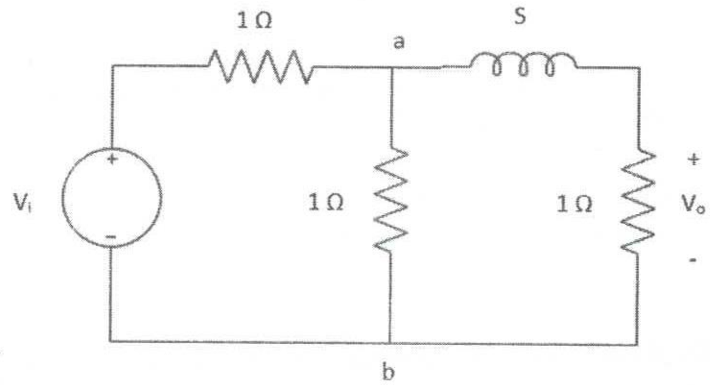


Fig. Q3.a

i) Find the transfer function $H(S) = V_o(S)/V_i(S)$.

[1 Mark]

ii) Find the impulse response.

[1 Mark]

iii) Find the output $v_o(t)$ when $v_i(t) = u(t)V$.

[1 Mark]

d) Consider the circuit diagram shown in Figure Q3.b. Assume that the initial conditions are zero.

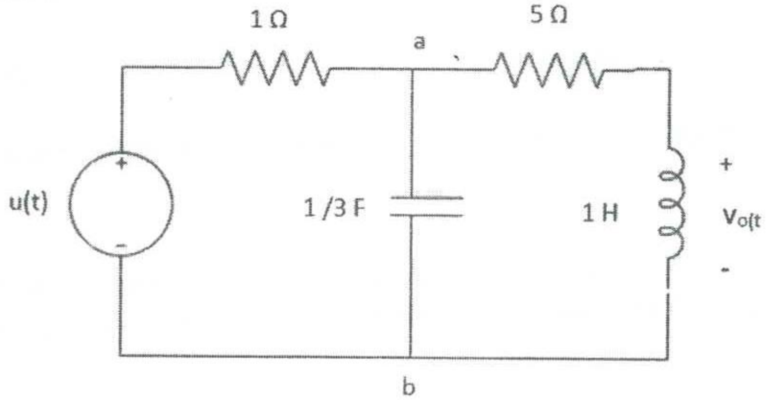


Fig. Q3.b

i) Draw the transformed circuit in s-domain by representing all the voltages and currents by their Laplace transforms.

[2 Marks]

ii) Determine the output voltage $v_o(t)$ in time domain.

[1 Mark]

**[Hint:** A capacitor C with zero initial condition can be represented by a capacitor of impedance $1/_{Cs}$ and similarly an inductor L with zero initial condition can be represented by an inductor of impedance $Ls$.]

Q4 a) "Spectral overlapping is unavoidable when sampling a practical signal regardless of the sampling frequency".

i) Briefly explain why spectral overlapping is unavoidable when sampling practical signals regardless of the sampling frequency.

[2 Marks]

ii) Explain how the spectral overlapping problem given in part a) i) can be eliminated.

[2 Marks]

b) The signal $f(t) = sinc^2 5\pi t$ shown in Fig. Q4.a. is sampled using an ideal sampling function. The frequency spectra of $f(t)$ is shown in Fig. Q4.b.

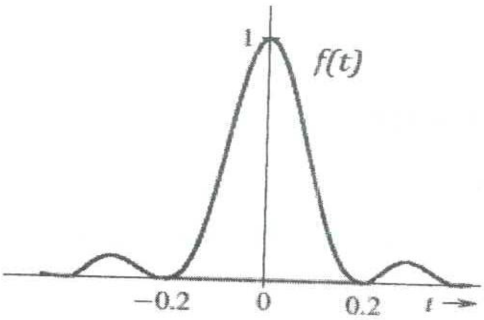


Fig. Q4.a

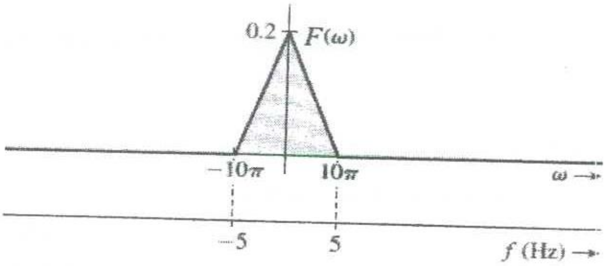


Fig. Q4.b

i) Determine the minimum satisfactory sampling frequency.

[2 Marks]

ii) Sketch the frequency spectra of the sampled signal if the sampling duration $T_s = 0.2s$. Clearly indicate the frequencies and amplitudes.

[2 Marks]

iii) Sketch the frequency spectra of the sampled signal if the sampling duration $T_s = 0.05s$. Clearly indicate the frequencies and amplitudes.

[2 Marks]

Q5 a) z - transform of a signal $f[n]$ is given by $F[z]$.

i) State a mathematical expression for $F[z]$.

[1 Mark]

ii) Determine the z-transform of the signal $f[n] = [3(2^n) - 4(3^n)]u(n)$ using the linearity property.

[2 Marks]

iii) Sketch the Region of Convergence (ROC) for the part a) ii).

[2 Marks]

b) Consider the discrete time signal $f[k]$ shown in the Fig.Q5.

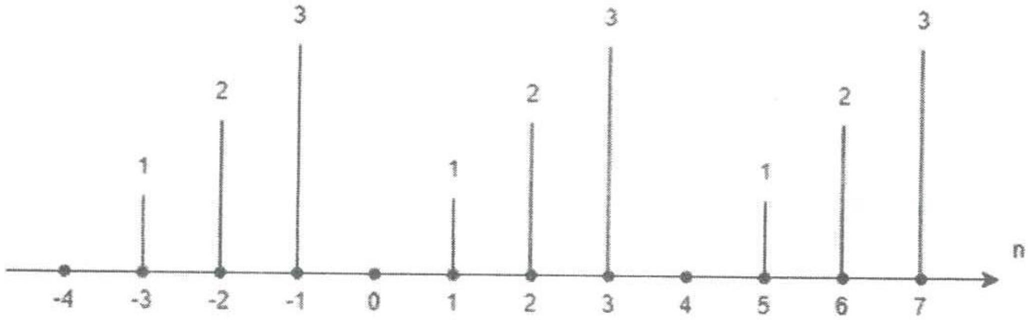


Fig.Q5

i) State a mathematical expression for the Discrete Time Fourier Series (DTFS) of a periodic signal $f[k]$ with the fundamental period $N_o$.

[1 Mark]

ii) Determine DTFS of the signal shown in Fig. Q5.

[2 Marks]

iii) Sketch the amplitude spectra and the phase spectra for the part b) ii).

[2 Marks]

Table 1: A short table of unilateral Laplace transforms

| | $f(t)$ | $F(s)$ |
|---|---|---|
| 1 | $\delta(t)$ | $1$ |
| 2 | $u(t)$ | $\frac{1}{s}$ |
| 3 | $tu(t)$ | $\frac{1}{s^2}$ |
| 4 | $t^n u(t)$ | $\frac{n!}{s^{n+1}}$ |
| 5 | $e^{\lambda t}u(t)$ | $\frac{1}{s-\lambda}$ |
| 6 | $te^{\lambda t}u(t)$ | $\frac{1}{(s-\lambda)^2}$ |
| 7 | $t^n e^{\lambda t}u(t)$ | $\frac{n!}{(s-\lambda)^{n+1}}$ |
| 8a | $\cos bt\, u(t)$ | $\frac{s}{s^2+b^2}$ |
| 8b | $\sin bt\, u(t)$ | $\frac{b}{s^2+b^2}$ |
| 9a | $e^{-at}\cos bt\, u(t)$ | $\frac{s+a}{(s+a)^2+b^2}$ |
| 9b | $e^{-at}\sin bt\, u(t)$ | $\frac{b}{(s+a)^2+b^2}$ |
| 10a | $re^{-at}\cos(bt+\theta)\, u(t)$ | $\frac{(r\cos\theta)s+(ar\cos\theta-br\sin\theta)}{s^2+2as+(a^2+b^2)}$ |
| 10b | $re^{-at}\cos(bt+\theta)\, u(t)$ | $\frac{0.5re^{j\theta}}{s+a-jb}+\frac{0.5re^{-j\theta}}{s+a+jb}$ |