UNIVERSITY OF RUHUNA

Faculty of Engineering

End-Semester 5 Examination in Engineering: May 2023

Module Number: ME5302 **Module Name: Computer Aided Design (C-18)**

[Three Hours]

[Answer **all** questions. All questions carry equal marks]

Q1. a) Write the three basic approaches to solve a finite element problem. Describe them briefly.

[3.0 Marks]

b) Figure Q1-b shows a thin plate which is subjected to a tensile stress as illustrated. Draw a neat sketch of suitable finite element mesh pattern by considering the symmetry of the model and specify the boundary conditions in the sketch. Show the transition zone in between fine mesh and coarse mesh clearly.

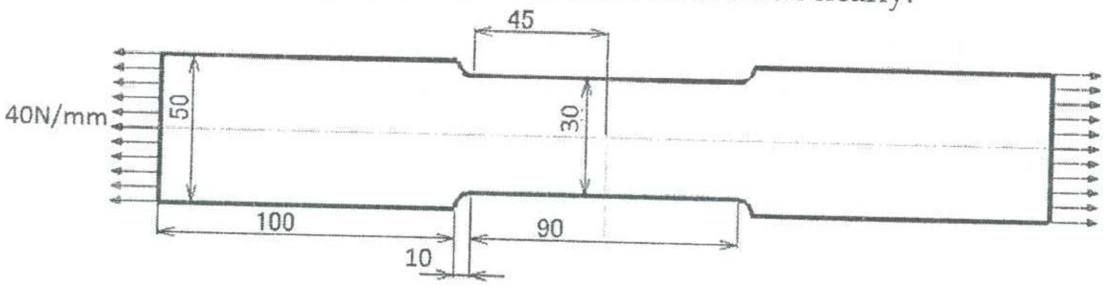

Figure Q1-b
All dimensions are in millimeters.

[5.0 Marks]

c) The spring assemblage with arbitrarily numbered nodes is given in Figure Q1-c. A force of 5000 lb is applied at node 4 in the x direction. The spring constants are given in the figure. Nodes 1 and 2 are fixed. Obtain the general equation for total potential energy of the spring assemblage and find its minimum value. Thus obtain,

(i) the global stiffness matrix,
(ii) the displacements of nodes 3 and 4,
(iii) the reaction forces at nodes 1 and 2,

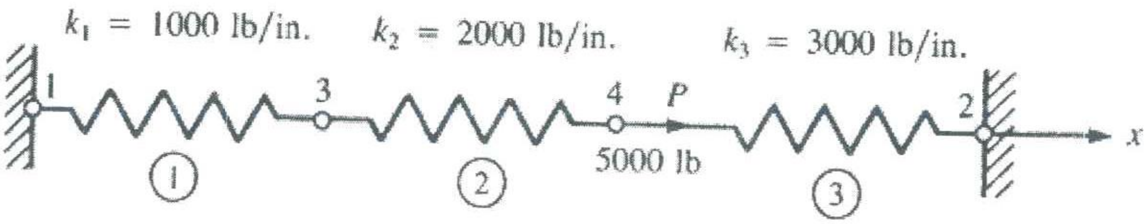

Figure Q1-c

[4.0 Marks]

Q2. a) List three types of coordinate systems which are needed in order to input, store and display model geometry and briefly describe them.

[3.0 Marks]

b) Name three important characteristics of CAD databases and explain them.

[3.0 Marks]

c) There are four types of database models used in CAD databases. Describe all four models one by one, by emphasizing the advantages and disadvantages.

[4.0 Marks]

d) Derive parallel line algorithm and find the initial decision parameter. Suppose $\Delta x=13$ and the number of processors are 4, find the width of the partitions and starting x values for the partitions. Consider the starting point as (x_0, y_0).

[2.0 Marks]

Q3 a) Derive homogeneous rotational transformation matrices for the rotation about X, Y and Z axes. Provide a clear figure of Cartesian coordinate system for each transformation. Use θ as rotation angle.

[3.0 Marks]

b) Derive a general homogeneous transformation matrix for scaling with respect to a selected fixed position (Xp, Yp, Zp). Use scale factors in X, Y and Z as Sx, Sy and Sz respectively. Provide all the steps clearly.

[4.0 Marks]

c) If you are asked to scale the object illustrated in Figure Q3-c by a scale factor of 2 in all axes with respect to a given point P (1,0,1), find the coordinates of the points A, B, C, D, and E of the figure after scaling up.

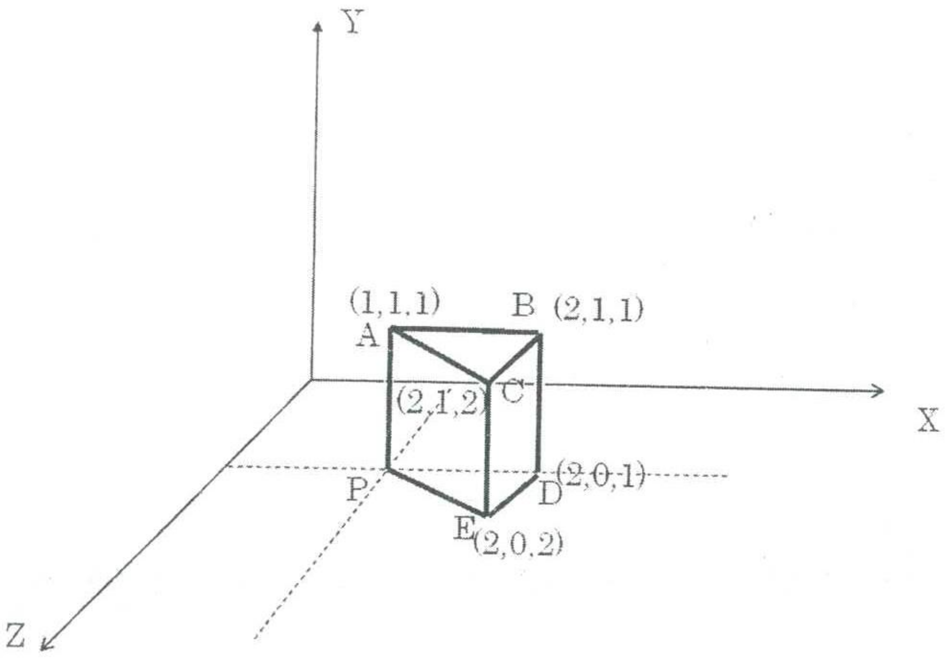

Figure Q3-c

[5.0 Marks]

Q4 a) Draw a closed Bezier curve with five control points. Draw its characteristic polygon and label the five control points.

[2.0 Marks]

b) Describe the effect on the quadratic Bezier curve when you make control points P_0 and P_2 exactly the same.

[2.0 Marks]

c) Suppose that you join two Bezier curves of degree 2, using the control point sequences (P_0, P_1, P_2) and (P_2, P_3, P_4), respectively. What conditions must be satisfied to have C^2 continuity at the joining point?

[3.0 Marks]

d) Express the PC curves in terms of the following geometric boundary conditions (starting point, mid-point, tangent at the mid-point and ending point).

You may use the following information if necessary:

$$\begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 2 & 0 & 0 \\ 6 & 2 & 0 & 0 \end{bmatrix}^{-1} = \begin{bmatrix} 0 & 0 & -1/6 & 1/6 \\ 0 & 0 & 1/2 & 0 \\ -1 & 1 & -1/3 & -1/6 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 3 & 2 & 1 & 0 \end{bmatrix}^{-1} = \begin{bmatrix} 2 & -2 & 1 & 1 \\ -3 & 3 & -2 & -1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 0 & 1 \\ 0.125 & 0.25 & 0.5 & 1 \\ 0.75 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} -4 & 0 & -4 & 4 \\ 8 & -4 & 6 & -4 \\ -5 & 4 & -2 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

[5.0 Marks]

Q5 a) Discuss the advantages of B-spline curves over Bezier curves for CAD applications.

[3.0 Marks]

b) Derive the equation for a linear B-spline defined by four control points.

[9.0 Marks]

Following equations are given.

$$p(u) = \sum_{i=0}^{n} p_i N_{i,k}(u)$$

$$0 \le u < (n+1) - (k-1) \qquad [(n+1) - (k-1)] > 0$$

$$N_{i,1} = \begin{cases} 1 & if\ t_i \le u < t_{i+1} \\ 0 & otherwise \end{cases}$$

$$N_{i,k}(u) = \frac{(u - t_i)N_{i,k-1}}{t_{i+k-1} - t_i} + \frac{(t_{i+k} - u)N_{i+1,k-1}}{t_{i+k} - t_{i+1}}$$

$t_i = 0 \quad if\ i < k$

$t_i = i - k + 1 \quad if\ k \le i \le n$

$t_i = n - k + 2 \quad if\ i > n$

Range of i $\ 0 \le i \le n + k$