UNIVERSITY OF RUHUNA

Faculty of Engineering

End-Semester 5 Examination in Engineering: May 2023

Module Number: CE 5205 **Module Name: Structural Analysis III**

[Three Hours]

[Answer **all questions**, each question carries twelve marks]

Q1. a) Yield line theory is considered as a very good analytical tool that can be used to assess the reinforced concrete slabs. What are the parameters required in strength assessment of reinforced concrete slabs by using yield line theory?

[3 Marks]

b) Figure Q1 shows an orthotropic reinforced concrete slab spanning over four beams at four edges. The yield moments per unit length of reinforcements, which are provided to resist sagging moment and hogging moment of the slab, are m and m', respectively. Yield moments for slab reinforcements are shown in Figure Q1. Yield moment for beam reinforcements is $2mL$. Assume that $m'=1.5m$, and $\mu=0.5$

i) Draw a possible yield line pattern at collapse.

ii) Determine the ultimate uniformly distributed load that can be carried by the slab.

[9 Marks]

Q2. a) State the basic assumptions of the bending theory of laterally loaded thin elastic plate with small deflections.

[3 Marks]

b) A thin rectangular plate having side dimension of '$2L$' and 'L' with thickness 't' is shown in Figure Q2. The plate is simply supported along all four edges and is subjected to a transverse load of intensity,

$$q(x,y) = q_o sin\frac{\pi x}{L} sin\frac{6\pi y}{L}$$

where, q_o is a constant.

i) Assuming a trail solution for the displacement, show that the trail solution satisfies the relevant displacement and boundary conditions.

ii) Determine expressions for deflection and bending moments of the plate. Hence determine the maximum deflection and the maximum bending stress of the plate.

The governing equation and the equations for bending moments (with usual notations and sign convention) for rectangular plates are given by

$$\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = \frac{q}{D} \qquad M_x = -D\left(\frac{\partial^2 w}{\partial x^2} + \nu\frac{\partial^2 w}{\partial y^2}\right) \qquad M_y = -D\left(\frac{\partial^2 w}{\partial y^2} + \nu\frac{\partial^2 w}{\partial x^2}\right)$$

where ,

$$D = \frac{Et^3}{12(1-\nu^2)}$$

[9 Marks]

Q3. a) Discuss with examples, the applications of thin circular plates in civil engineering structures.

[3 Marks]

b) A circular plate of radius "a" and uniform thickness" t" is simply supported along the edge and is subjected to a uniformly distributed load "p"/ unit area. Young's Modulus of elasticity and Poisson ratio of plate materials are E and ν, respectively.

i) Determine the maximum deflection of the plate.
ii) Determine the maximum bending stress.
iii) Evaluate the required minimum thickness of a plate having a diameter of 8 m and carrying uniformly distributed load of 10 kN/m2, if the deflection of the plate is limited to 70 mm. Assume that Young's Modulus of elasticity is 210 GPa and Poisson ratio is 0.2.

Governing equation and the equation for the radial moment of circular plate (with usual notations and sign convention) are given by

$$\frac{d}{dr}\left[\frac{1}{r}\frac{d}{dr}(r\frac{dw}{dr})\right] = \frac{Q}{D} \qquad M_r = -D\left(\frac{d^2 w}{dr^2} + \frac{\nu}{r}\frac{dw}{dr}\right) \qquad M_t = -D\left(\frac{1}{r}\frac{dw}{dr} + \nu\frac{d^2 w}{dr^2}\right)$$

where

$$D = \frac{Et^3}{12(1-\nu^2)}$$

[9 Marks]

Q4. a) "Closed surfaces are rigid". Do you agree with this statement? Justify your answer providing examples of natural shell structures.

[3 Marks]

b) A roof is formed by a spherical shell having a radius "a". The roof has a circular base of radius" $a/\sqrt{2}$ "and is subjected to a uniformly distributed 'snow-load' of "f"/per unit surface area.

i) Determine membrane stresses caused by snow load.
ii) Plot the variation of membrane stresses along the height of the roof. Clearly indicate the magnitude of stresses in tension and compression.

Membrane stresses in a spherical shell (with usual notations and sign convention) are given by

$$\frac{N_\phi}{r_1} + \frac{N_\theta}{r_2} = P_r \qquad P_\phi r r_1 - r_1 N_\theta Cos\phi + \frac{\partial(rN\phi)}{\partial\phi} = 0$$

[9 Marks]

Q5. a) Show that the membrane stresses in a conical shell (*with usual notations and sign convention*) are given by

$$N_\theta = P_r\, S \, \tan\alpha$$

$$N_S = \frac{1}{S}\int (P_r\, S\, \tan\alpha - P_s\, S)ds$$

[6 Marks]

b) A roof of a building is proposed to be constructed with a conical shell structure. The conical shell with an apex angle of "α" is extending from the apex to a base circle of radius "b". The shell is suspended from its apex with its vertical axis, and is loaded by a self-weight of intensity "f"/per unit plan area. Determine the membrane stresses on the conical shell structure.

[6 Marks]

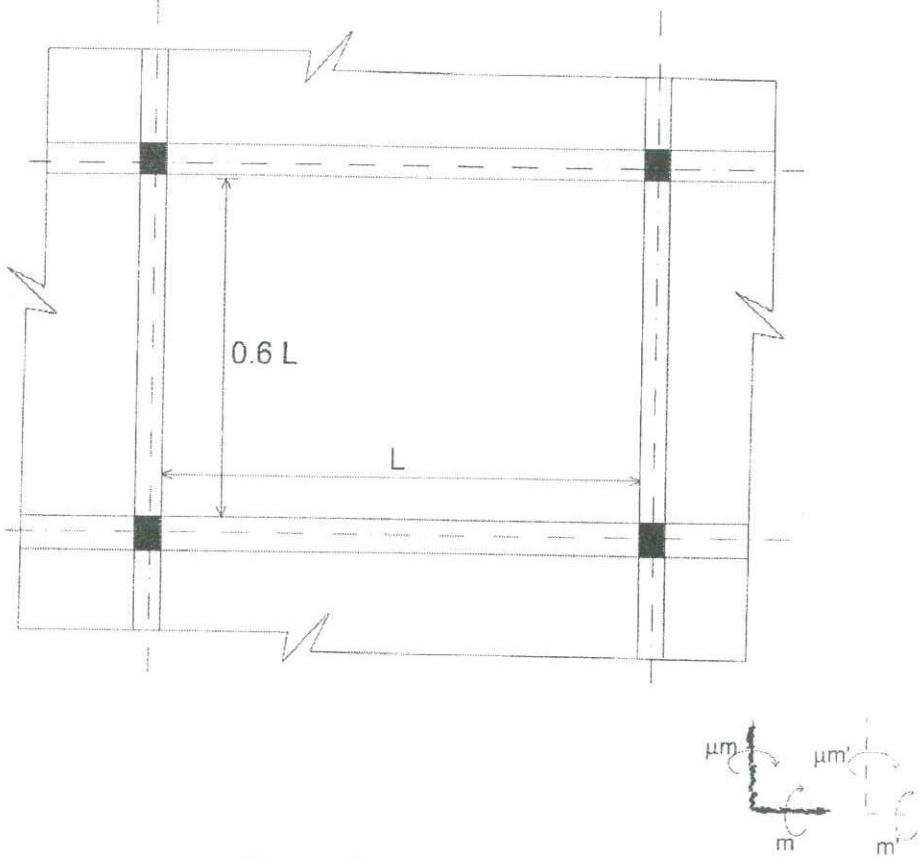

Figure Q1

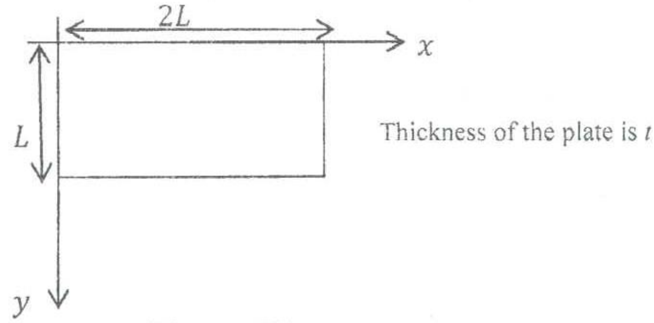

Figure Q2