UNIVERSITY OF RUHUNA

Faculty of Engineering

End-Semester 6 Examination in Engineering: November 2022

Module Number: ME6213 **Module Name: Robot Manipulator Kinematics**

[Three Hours]

[Answer **all questions**, each question carries **12** marks]

This paper contains **5 questions** on 6 **pages**.
Clearly state any assumptions that you may make.
In order to get **full marks**, make sure to use standard notations and SI units.

Q1. (a) Briefly describe four industrial applications of robot manipulators.

[2.0 Marks]

(b) Define the terms "redundancy" and "Degree of redundancy" related to robot manipulators.

[2.0 Marks]

(c) Assume you are an automation engineer tasked with automating a defect item removal operation in a product assembly line using a robot manipulator to replace a human worker. The layout of the operation is shown in Figure Q1(c).

(i) Compare and contrast the use of a robot manipulator over a human worker when performing a pick-and-place task.

(ii) Explain the importance of determining the best robot parameters before purchasing a robot manipulator for this application in terms of quality of the work and production rate.

(iii) Decide a suitable type of a robot manipulator for this application and state three reasons for your selection.

[8.0 Marks]

Q2. (a) The coordinate frame {B} is free to translate and rotate with respect to a fixed coordinate frame {A} as shown in Figure Q2(a). ${}^{A}R_{B}$ and ${}^{B}R_{A}$ are representing the basic rotational matrices with usual notation.

(i) Obtain ${}^{A}R_{B}$ and ${}^{B}R_{A}$ using scalar products of vector components method.

(ii) Show that rotational matrices (${}^{A}R_{B}$, ${}^{B}R_{A}$) are orthonormal.

(iii) Show that ${}^{A}P = {}^{A}R_{B}\,{}^{B}P + {}^{A}d_{B}$ and derive the homogeneous transformation matrix (${}^{A}T_{B}$).

[8.0 Marks]

(b) The planar robot arm shown in Figure Q2(b) rotates about the Z_a axis. Find the homogeneous transformation matrix (${}^{a}T_{b}$).

[4.0 Marks]

Q3. A 3-DOF SCARA type robot manipulator is shown in Figure Q3.

(a) Draw a suitable sketch and state four *Denavit–Hartenberg* (D-H) parameters clearly.

[2.0 Marks]

(b) Sketch the robot skeleton diagram and assign coordinate frames for the links of the given robot (Figure Q2) based on D-H representation.

[2.0 Marks]

(c) Obtain the DH parameters for this robot manipulator in the form of a table.

[3.0 Marks]

(d) Derive individual homogeneous transformation matrices for the robot.

[3.0 Marks]

(d) Determine the transformation matrix for the tool-frame coordinates w.r.t. the base using the results of above Q3(d).

[2.0 Marks]

Q4 (a) Briefly explain importance of the Jacobian in the derivations of robot manipulator kinematics.

[1.0 Mark]

(b) Given that a scalar function $y = f(x)$ where $x = [x_1 \quad x_2 \cdots x_n]^T$ and $y = [y_1 \quad y_2 \cdots y_n]^T$, define the Jacobian (J).

[1.0 Mark]

(c) A 2 link revolute joint manipulator is shown in Figure Q4(c).

(i) Obtaion the relationship between the end effector velocity and the joint velocities.

(ii) Mathematically define the singular Jacobian and Non-sigular Jacobian.

(iii) Obtain the singular points for the 2 link manipaulator above and demonstrate the meaning of "Losing a DoF" at a singular point using suitable sketches.

[4.0 Marks]

(d) The velocity of link i shown in Figure Q4(d) by vectors v_i and ω_i, which may be written in any frame $\{i\}$. Consider both joints are as revolute joints.

(i) Copy this diagram to your answer script and draw the velocity vectors of a neigboring link taking it as the frame $\{i+1\}$.

(ii) Show that the velocities of link $i+1$ are given by

$$^{i+1}\omega_{i+1} = {}^{i+1}_{i}R \; {}^{i}\omega_i + \dot{\theta}_{i+1} \, {}^{i+1}\hat{Z}_{i+1}$$

$$^{i+1}v_{i+1} = {}^{i+1}_{i}R \; ({}^{i}v_i + {}^{i}\omega_i \times {}^{i}P_{i+1})$$

Note: All the parameters are in standard notations.

[4.0 Marks]

(e) Calculate the velocity of the tip of the arm of the two link manipulator robot shown in Figure Q4(e) as a function of joint rates ($\dot{\theta}_1$ and $\dot{\theta}_2$) with reference to frame $\{3\}$.

[2.0 Marks]

Q5 (a) Describe the difference between the path and the trajectory of a robot manipulator using suitable sketches.

[2.0 Marks]

(b) Describe and compare the following terms with related to robot trajectory planning.

(i) Joint space

(ii) Cartesian space

[3.0 Marks]

(c) It is required to have the first joint of a 6-axis robot go from an initial angle of 30° to a final angle of 75° in 5 seconds. Using a third-order polynomial, calculate the following at 1, 2, 3, and 4 seconds.

(i) The joint angle

(ii) The joint velocity

(iii) The joint acceleration

[3.0 Marks]

(d) Suppose the robot arm in above (c) is to continue to the next point, where the joint is to reach 105° in another 3 seconds. Draw the position, velocity, and acceleration curves for the motion for the whole period (i.e. 0 to 7 seconds).

[2.0 Marks]

(e) Beifly explain the following terms with related to robot manipulator use. Give one example for each.

(i) Dexterous workspace

(ii) Reachable workspace

[2.0 Marks]

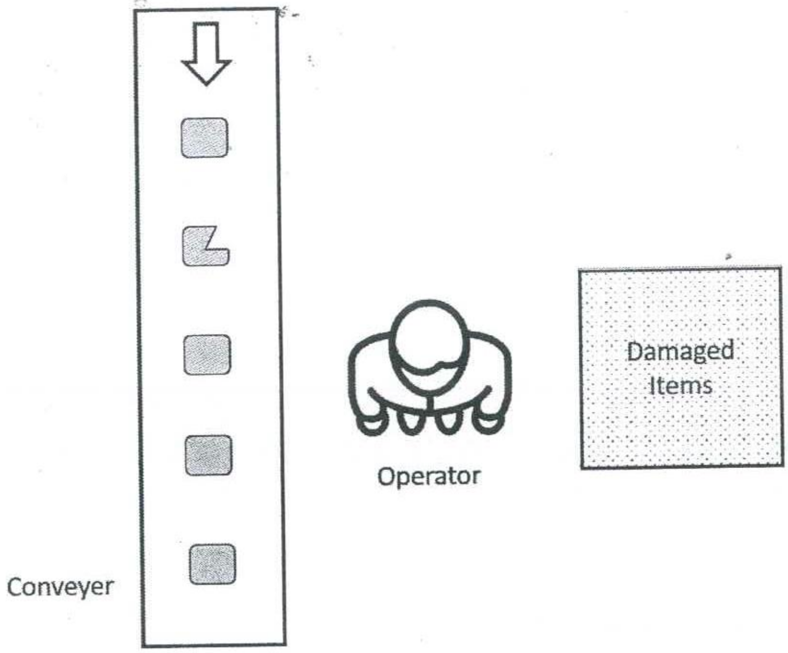


Figure Q1(c) - Layout of the defect item removal operation

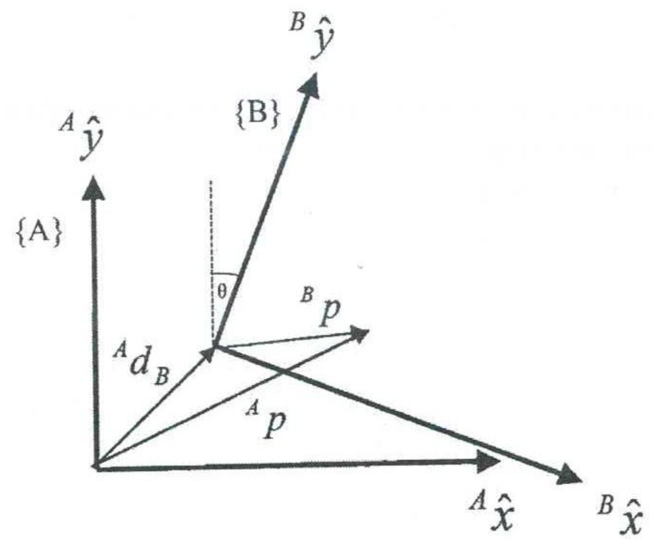


Figure Q2(a) – {A} and {B} coordinate systems

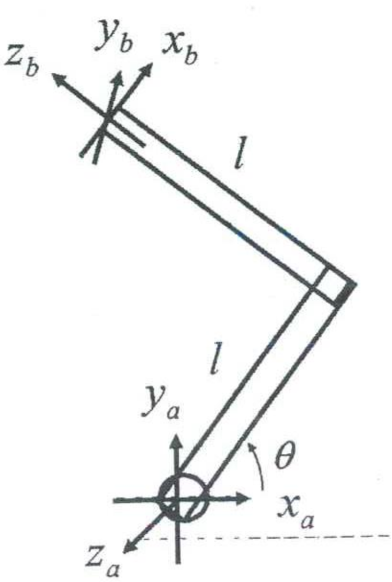
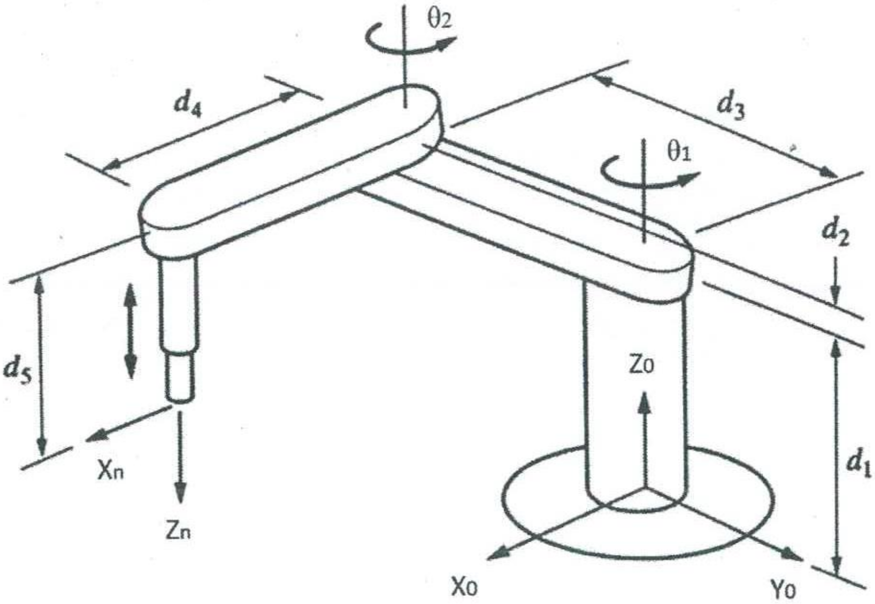


Figure Q2(b) – Planar robot arm (all revolute joints)



$$T = \begin{bmatrix} \cos\theta_i & -\sin\theta_i \cos\alpha_i & \sin\theta_i \sin\alpha_i & a_i \;\; \cos\theta_i \\ \sin\theta_i & \cos\theta_i \cos\alpha_i & -\cos\theta_i \sin\alpha_i & a_i \;\; \sin\theta_i \\ 0 & \sin\alpha_i & \cos\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Figure Q3 – 3 DOF SCARA Robot

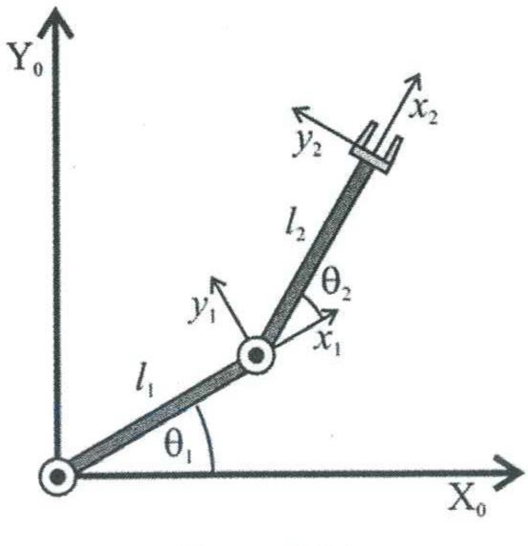


Figure Q4(c)

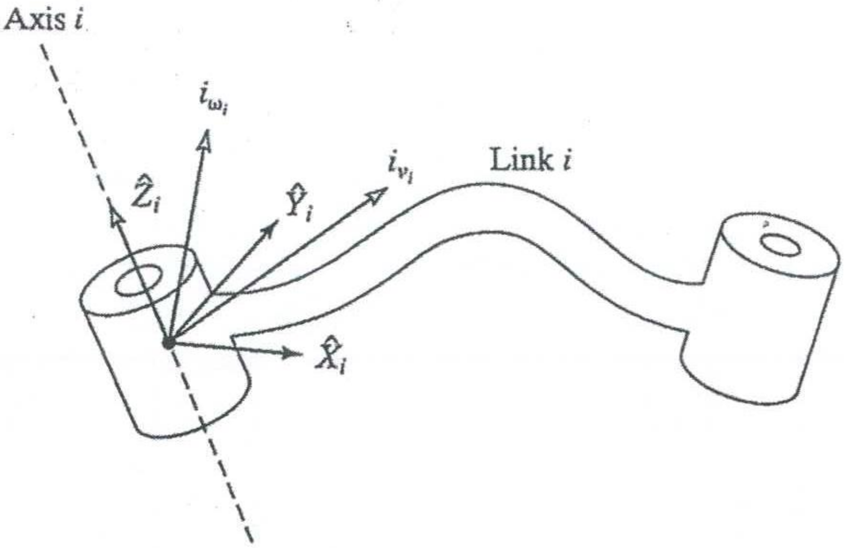


Figure Q4(d)

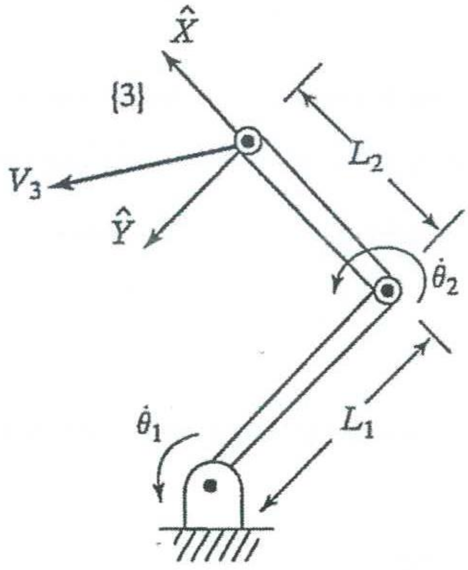


Figure Q4(e)