UNIVERSITY OF RUHUNA

Faculty of Engineering

End-Semester 6 Examination in Engineering: November 2022

Module Number: ME 6206 **Module Name: Solid Mechanics**

[Three Hours]

[Answer **all questions**, each question carries **12** marks]

Note: Clearly state any assumptions made in answering the questions. Use usual notations.

Q1. a) The forces due to the pressure of the fluid acting at the ends of the thin cylinder tend to burst the cylinder. The bursting will take place if the force due to fluid pressure is more than the resisting force due to circumferential stress set up in the material. Considering the limiting case, derive expressions for circumferential stress and longitudinal stress acting on a thin cylindrical shell due to the fluid pressure.

[2.0 Marks]

b) Consider a thin cylindrical shell subjected to internal fluid pressure. The effect of the lateral strain is caused by some changes in the dimensions of the length and diameter of the shell. Derive an expression for the change in volume of the cylinder and hence, show that the change in volume depends on the original dimensions of the cylinder and internal fluid pressure.

[6.0 Marks]

c) A spherical shell of internal radius and shell thickness are 1.0 m and 20 mm, respectively, is subjected to the internal fluid pressure of 5.5 N/mm^2. The joint efficiency of the shell material is 75%.

i) Calculate the stress induced in the shell material as a result of internal fluid pressure.

[2.0 Marks]

ii) Find the change of stress in the shell material, if the joint efficiency is increased to 90%.

[2.0 Marks]

Q2. a) Briefly explain **four (04)** applications of rotating discs.

[2.0 Marks]

b) A thick cylindrical pipe with an outside diameter of 300 mm and internal diameter of 200 mm is subjected to an internal fluid pressure of 14 N/mm^2. Determine the maximum hoop stress developed in the cross-section. What is the percentage error if the maximum hoop stress is calculated by the equations for a thin cylinder?

[8.0 Marks]

c) The cylinder of a hydraulic jack has a bore of 150 mm and is required to operate up to 13.8 MN/m^2. Determine the required wall thickness if the limiting tensile stress of the material is 41.4 MN/m^2.

[2.0 Marks]

Q3. a) Briefly explain, using a figure, the approximations used in stress strain diagram in order to solve plastic deformation problems.

[2.0 Marks]

b) Draw a figure of a tensile failure of a failed specimen in a simple tensile test. Mark down the different failure regions.

[2.0 Marks]

c) Consider a metal beam with a rectangular cross section as shown in Figure Q3 (c). This metal beam is subjected to a pure bending moment such as the plastic deformation is penetrated into the beam. Plastic deformation has penetrated into the beam a distance of d/4 from one side.

Note : Same amount of plastic deformation penetration should be present from the opposite side too. Bending moment is applied to the beam such that, topmost surface and bottommost surfaces will receive the maximum strain.

Take : Yield stress of the beam is 240 MN/m^2.
Dimensions of the beam d = 15 cm and b = 10 cm.

i) Draw the stress distribution of the beam with the plastic deformation along the y axis.

[1.0 Mark]

ii) Calculate the bending moment acting on the beam.

[1.0 Mark]

iii) Calculate the maximum pure bending moment that can be supported by the beam.

[1.0 Mark]

iv) Calculate the shape factor of the beam.

[1.0 Mark]

Data given:

$$M_e = br^2\sigma_y(\varepsilon_y^2 - \frac{1}{3}\varepsilon_y^2)$$

$$M = br^2\sigma_y(\varepsilon_1^2 - \frac{1}{3}\varepsilon_y^2)$$

$$\varepsilon_1 = \frac{h}{2r}$$

where,
M_e – Bending moment at elastic condition
M – Bending moment during bending beyond the yield point
b, d – parameters from the figure (width and high of the beam)
r – radius of curvature
σ_y – Yield stress
ε_y – Strain at the yield stress
ε_1 – Strain at outermost fibers during bending beyond the yield point

d) A propeller shaft of a ship should be transmitted 15000 Nm torque to the propeller. The shaft has a diameter of 80 mm and a yield shear of 120 MN/m^2.

i) From which surface the plastic deformation starts on the shaft?

[1.0 Mark]

ii) Calculate the distance of plastic deformation penetrated into the shaft.

[1.0 Mark]

iii) Calculate the minimum diameter of the shaft if the shaft is to remain in fully elastic condition when subjected to 15000 Nm.

[1.0 Mark]

iv) Calculate the maximum torque that the 80 mm shaft can withstand.

[1.0 Mark]

Data given:

$$M = \int \tau \times 2\pi r \times dr \times r$$

$$\tau = \gamma G = r\theta G$$

where,

M – Torque on the shaft
τ – Shear stress at the considered point
r – Radius at the considered point
γ – Shearing strain
θ – Angle of twist per unit length

Q4. Even though, there are numerous instances where we assume plain stress-strain conditions with sufficient reasoning accompanied by a good engineering judgment to reduce the complexity in analyzing, the stress state at a point must always be considered as three-dimensional (3D), unless specified explicitly.

a) Using standard notations, draw the 3D stress element (use the Figure Q4 (a)) and specified coordinate system) marking all stress components to fully describe the state of stress at a point and obtain the stress tensor.

[2.0 Marks]

b) The fully described 3D state of stress conditions, using suitable notations, show that the complementary shear concept holds true.

$\boldsymbol{\tau_{xy} = \tau_{yx}}$, $\boldsymbol{\tau_{xz} = \tau_{zx}}$, and $\boldsymbol{\tau_{yz} = \tau_{zy}}$

[2.0 Marks]

c) The stress state of a structural member in a particular machine is represented as shown in Figure Q4(c).

i) Draw the 3D element showing all the stress components.

[2.0 Marks]

ii) Find the maximum in-plane shear stress in **xz plane.**

[2.0 Marks]

iii) Find the absolute maximum shear stress.

[2.0 Marks]

iv) If the material of the particular structural member has a critical compression strength of 45 MPa and a yield strength of 235 MPa, comment on the stability of the particular machine part, basically explaining how it would behave under tension and compression conditions here.

[2.0 Marks]

Q5. An electrical strain gauge comprises of a highly sensitive resistive foil (would react even for the slightest changes in environmental conditions). Changes in the foil dimensions under different loadings can be converted to a strain measurement eventually.

a) How would the changes in temperature affect the accuracy of the strain measurements?

[2.0 Marks]

b) How can we use half-bridge and full-bridge configurations specifically to minimize the temperature effect and get the stain measurements as accurate as possible? Briefly explain with the aid of the sketches.

[2.0 Marks]

c) A delta strain rosette (0^0, 60^0, 120^0) (Figure Q5(c)) is used in obtaining strain measurements experimentally in order to evaluate a design against its simulated performances. The strain rosette is bonded to the surface of a thin steel plate under the operating loading conditions.
The recorded strain measurements are as follows.

$\varepsilon_a = 50\mu\varepsilon$, $\varepsilon_b = 130\mu\varepsilon$, $\varepsilon_c = 255\mu\varepsilon$

i) Derive expressions for ε_x, ε_y, and γ_{xy} in terms of ε_a, ε_b and ε_c (assume plane strain conditions).

[1.0 Mark]

ii) Analyze the strain state to obtain the principle strains and their planes.

[4.0 Marks]

iii) Calculate the principle stresses (take Young's modulus, E = 225 MPa and poissons ratio, $\mu = 0.31$).

[1.5 Marks]

iv) Determine the plane strain conditions of x^1y^1 plane (in Figure Q5(c))

[1.5 Marks]

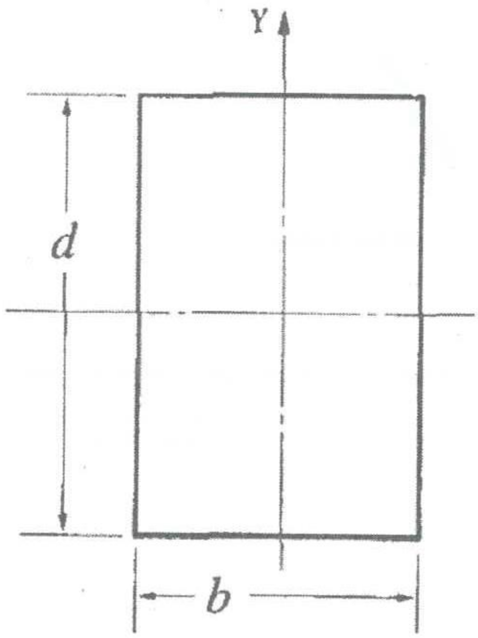


Figure Q3 (c)

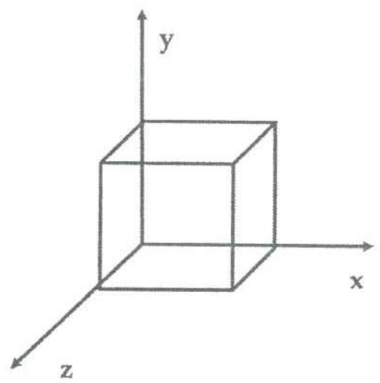


Figure Q4(a)

$$\begin{bmatrix} 20 & -40 & 0 \\ -40 & -40 & 0 \\ 0 & 0 & 100 \end{bmatrix}$$

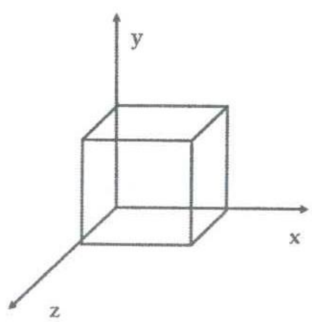
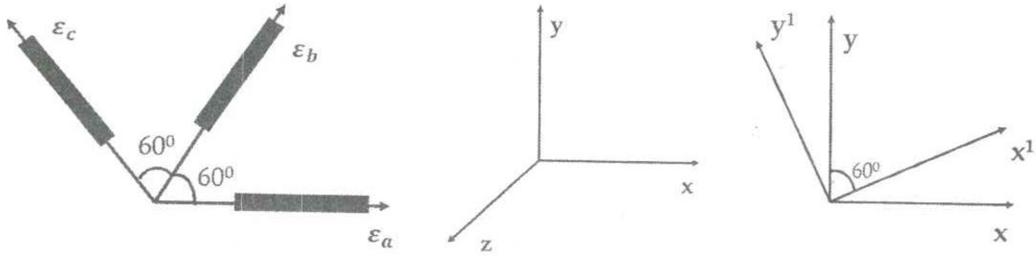


Figure Q4(c)



ε_c
ε_b
60^0
60^0
ε_a
y
x
z
y^1
y
x^1
60^0
x

Figure Q5(c)