University of Ruhuna- Faculty of Technology

Bachelor of Engineering Technology Honours Degree

Level 1 (Semester II) Examination, Nov-Dec. 2023

Academic Year 2021/2022

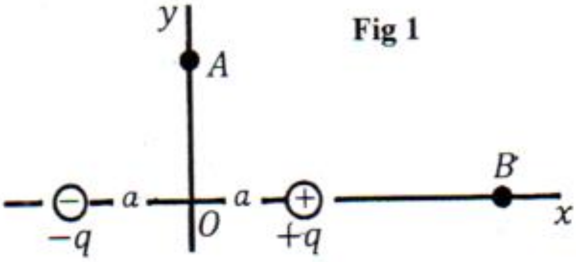
Course Unit: **ENT1242 Electricity and Magnetism** Duration: **2 hours**

Instructions and details:

- Answer all **Five (05)** questions.
- The paper carries a total of 100 marks and each question carries 20 marks.
- Calculators are allowed for calculations.
- When relevant, answers should be expressed in terms of the given (relevant) variables and simplified.
- All symbols have their usual meanings.
- $k = k_e = \frac{1}{4\pi\varepsilon_0} = 9 \times 10^9\ \mathrm{Nm^2/C^2}$.
- $\varepsilon_0 = 8.85 \times 10^{-12}\ \mathrm{C^2/Nm^2}$.
- $\mu_0 = 4\pi \times 10^{-7}\ \mathrm{T.m/A}$.

1. As shown in Figure 1, an electric dipole is made of two charges of equal magnitude q and opposite signs that are separated by a distance $2a$. The dipole is located along the x-axis and it is centered at the origin O of the xy coordinate system. Let k denote the Coulomb's constant.

Fig 1



(a) Find the electric field ($\vec{E}$) of the dipole at point $A(0, y)$ on the y-axis, in unit-vector notation. (*Hint/s: You may want to draw the fields. The field due to a point charge: $\vec{E} = k\frac{|q|}{r^2}\hat{r}$)

(b) Find the electric potential (V) at point $B(x, 0)$ on the x-axis as shown in the Figure.

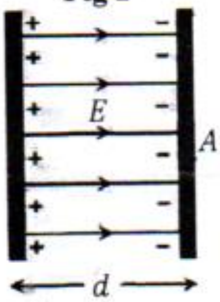
If the point $B(x, 0)$ is located far-away from the dipole (i.e. $x >> a$) then,

(c) Find the approximate potential at point B. [*Hint/s: part-(b)]

(d) Find the approximate electric field at point B in the positive x direction. [*Note/s: Use the result of part-(c). $\frac{d}{dx}x^n = nx^{n-1}$]

2.

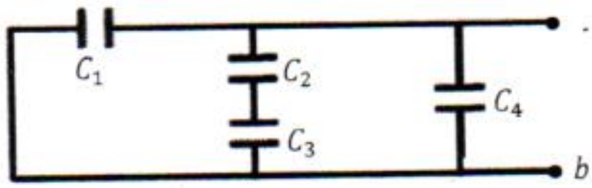
Fig 2



(i) In Figure 2, a charged parallel-plate capacitor is made of plates of area A and plate separation d. If the uniform electric field between the plates is $E = \sigma/\varepsilon_0$ then, find the capacitance (C) of the capacitor in terms of ε_0, A and d, where σ is the magnitude of the surface charge density of each plate. (*Hint/s: First you may want to show that the potential difference between the plates is $|\Delta V| = Ed$, and then consider the definition of capacitance using charge Q.)

(ii) Capacitors of capacitances $C_1 = C_2 = 20\ \mu\text{F}$ and $C_3 = C_4 = 30\ \mu\text{F}$ are connected in a capacitor network as shown in Figure 3.

Fig 3



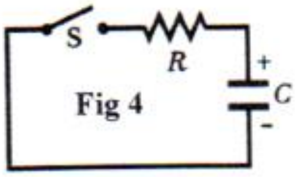
A potential difference of 10 V is applied between the points a and b. Then,

(a) Calculate the equivalent capacitance (C_{eq}), between the points a and b.

(b) Calculate the energy (U) stored in the capacitor network.

(iii) An RC series circuit is shown in Figure 4. The initial charge in C is Q_0. The switch S is turned on at time $t = 0$, so that C discharges. Then, show that the charge of the capacitor at time t can be written as

$$q = Q_0 e^{-\frac{t}{RC}}.$$



(*Hint/s: Current $i = -\frac{dq}{dt}$. The solution for the differential equation $\frac{dq}{dt} + bq = 0$ can be written as $q = ae^{-bt}$, where a and b are constants.)

3. In the circuit in Figure 5, the resistances of the resistors are $R_1 = R_5 = 200\ \Omega$, $R_3 = 400\ \Omega$, $R_2 = R_4 = 800\ \Omega$ and the ideal emfs of the batteries are $\varepsilon_1 = 5.00$ V and $\varepsilon_2 = 10.0$ V.

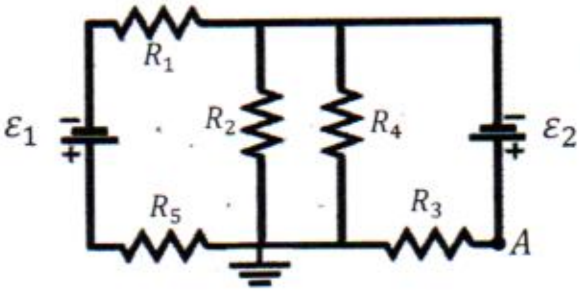


Fig 5

(a) Calculate the current flowing through R_1. (*Hint/s: first you may want to reduce, simplify & redraw the circuit. Kirchhoff's rules.)

(b) Calculate the current flowing through R_2.

(c) Calculate the current flowing through R_3.

(d) Mention whether the battery ε_2 is being charged or discharged. Briefly give your reasons.

(e) If one point of the circuit is grounded as shown then, calculate the electric potential at point A.

(f) Calculate the power dissipated by R_3.

(g) Calculate the potential difference across R_2.

4.

(i) An ideal long solenoid inductor has n number of turns per unit length and carries a current i. Then,

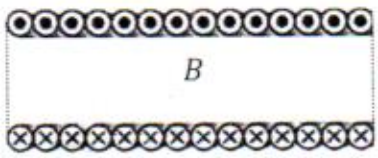


Fig 6

(a) Show that the uniform magnetic field generated inside the solenoid can be written as

$B = \mu_0\, i\, n.$ Note/s: See Figure 6. Use Ampere's law.

(b) If the solenoid has a length l and a circular cross-sectional area A then, find its inductance (L).

(c) If a straight wire of length l_0 carrying a current i is placed at the center of the solenoid perpendicular to its central axis then, find the magnitude of the magnetic force (F) on the wire.

(ii) A uniform magnetic field of magnitude B is perpendicular to the plane of a circular wire loop of radius r. If the loop has a resistance R then, find the rate at which B must be changed to induce a current i in it. [i.e. find $\left|\frac{dB}{dt}\right|$. *Hint/s: Ohm's law, Faraday's law]

(iii) A current i flows through a 10 H inductor. If an emf of 50 V is produced across it then, calculate the rate of change of the current. [i.e. calculate $\left|\frac{di}{dt}\right|$.]

5.

(i) As shown in Figure 7, an inductor $L = 6.0\ \mu\text{H}$ is connected in series with a resistor $R = 1.0\ \text{k}\Omega$. The emf of the battery is $\varepsilon = 12$ V. The switch S is turned on at time $t = 0$. Then, for the circuit

(a) Find the time constant. (i.e. the inductive τ)

(b) Find the current (i) at time $t = 6.0$ ns. [*Hint/s: $i = i_{max}\left(1 - e^{-\frac{t}{\tau}}\right)$, where $e = 2.72$]

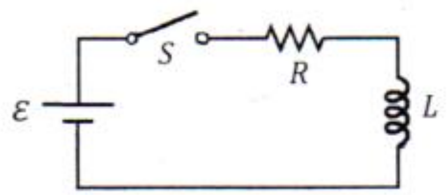


Fig 7

(ii) In Figure 8, an RLC circuit is made of a resistor $R = 180\ \Omega$, a capacitor $C = 100\ \mu\text{F}$, an inductor $L =$ 240 mH, and a driving emf ε that are connected in series. ε has an amplitude of $\varepsilon_m = 40.0$ V and a frequency of $f_d = 50.0$ Hz.

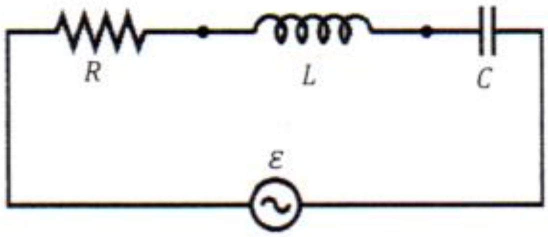


Fig 8

For the circuit,

(a) Calculate the reactance of C. [*Hint/s: $X_L = \omega_d L$, $X_C = 1/(\omega_d C)$]

(b) Calculate the reactance of L.

(c) Calculate the impedance (Z).

(d) Calculate the phase angle (ϕ).

(e) Calculate the RMS current. [*Hint/s: You may want to first calculate the current amplitude]

(f) Calculate the average rate at which energy is dissipated.

(g) Calculate the power factor.

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