UNIVERSITY OF RUHUNA

Faculty of Engineering

End-Semester 8 Examination in Engineering: September 2023

Module Number: EE 8204 **Module Name: Digital Communication**

[Three Hours]

[Answer all questions, each question carries 10 marks]
[Notations and symbols have their usual meaning unless otherwise stated]

Q1 a) Figure Q1(a) represents four signals A, B, C, and D corresponding to different angle modulations schemes: QPSK, OQPSK, CPFSK and MSK. Associate each signal with its corresponding modulation scheme and justify your answer.

[2 marks]

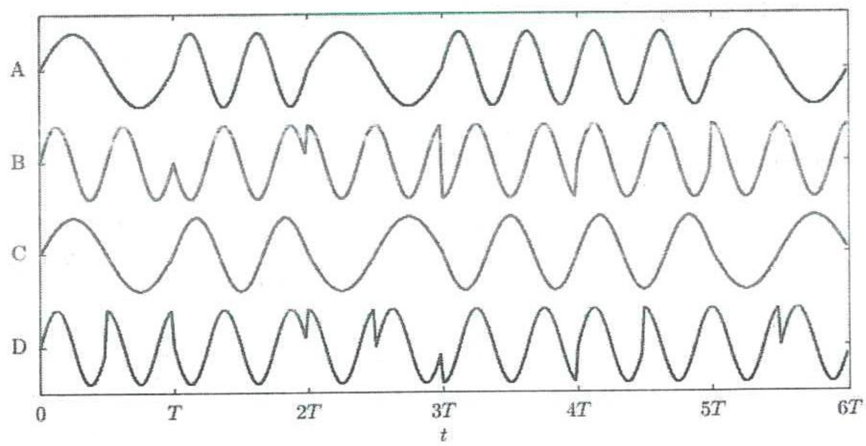

Figure Q1(a)

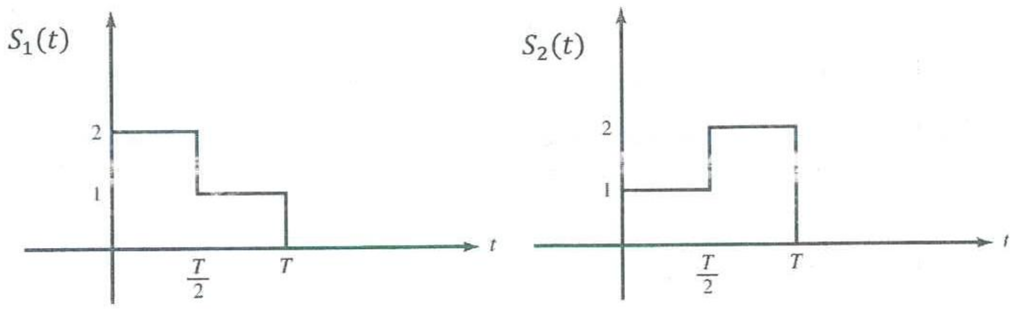

Figure Q1(b)

b) A binary signaling scheme over an AWGN channel with noise power spectral density of $\frac{N_0}{2}$ uses the equiprobable messages shown in Figure Q1(b). It is operating at a bit rate of R bits/s. What is $\frac{\mathcal{E}_b}{N_0}$ for this system (in terms of N_0 and R)? Here $\mathcal{E}_b$ denotes the energy per bit.

[2 marks]

c) Consider a digitally modulated binary PAM signal and refer to its power spectrum provided in Figure Q1(c).

i) Calculate the bandwidth efficiency of this system when 95% of signal power needs to pass through the channel.

ii) If we instead use a 4-ary PAM modulation scheme, determine the expected bandwidth efficiency required to maintain 95% of the in-band power.

[3 marks]

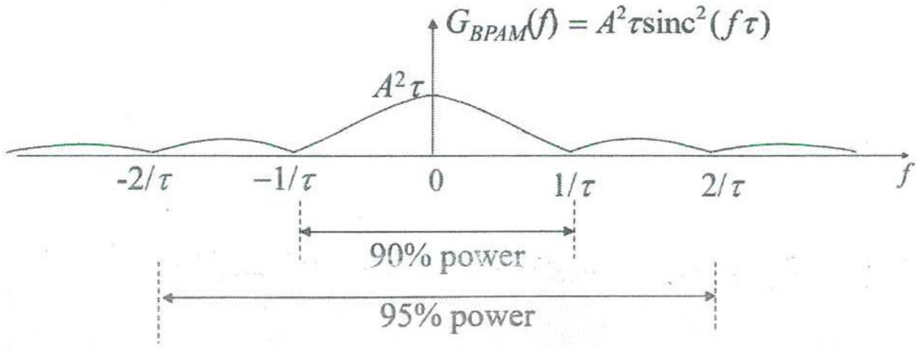

Figure Q1(c)

d) i) What is the rationale behind designing signals at baseband (lower frequencies) instead of being directly designed at higher frequencies?

ii) What necessitates the transmission of signals at bandpass frequencies, or high frequencies?

[3 marks]

Q2 The four signals shown in Figure Q2 are used for communication of four equiprobable messages over an AWGN channel. The noise power spectral density is $\frac{N_0}{2}$.

a) Find an orthonormal basis, with lowest possible dimensionality, for representation of the signals.

[3 marks]

b) Plot the constellation, and using the constellation, find the energy in each signal.

[2 marks]

c) Calculate the average signal energy and energy per bit?

[2 marks]

d) On the constellation that you have plotted, determine the optimal decision regions for each signal, and determine which signal is more probable to be received in error.

[3 marks]

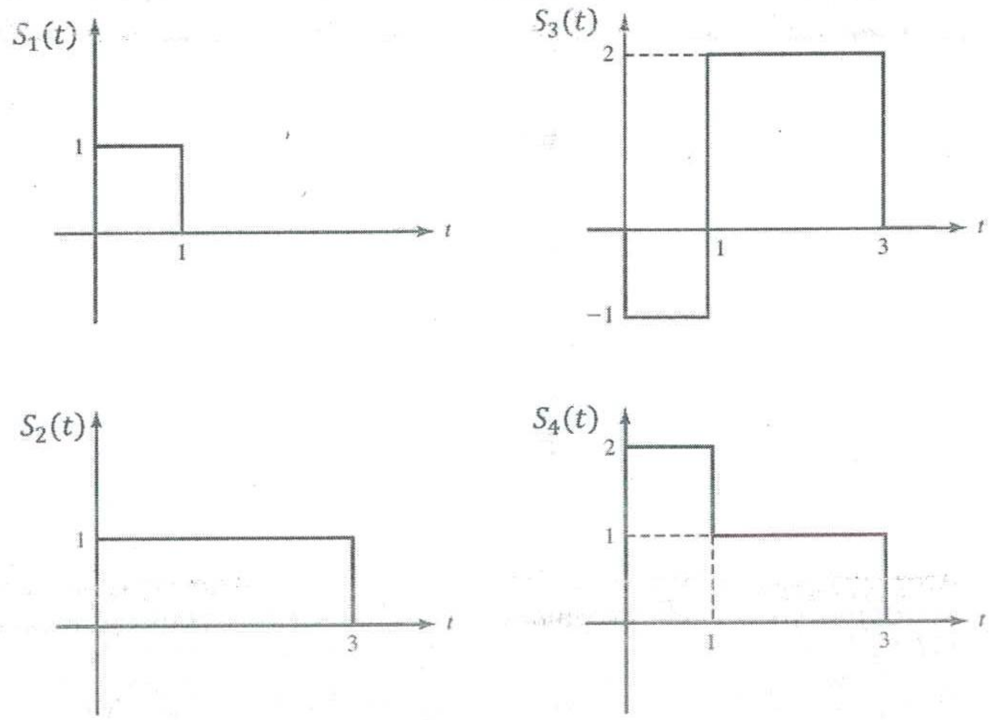

Figure Q2

Q3 a) i) How does the inter-symbol interfere (ISI) occur in digital communication systems?

ii) Briefly mention what is meant by adaptive channel equalization in wireless communications.

iii) What are the main reasons for the frequent preference of nonlinear equalizers in wireless communication applications?

[4 marks]

b) "In the process of removing ISI, the noise power in the received signal is also enhanced." Justify the above statement using a block diagram of a linear equalizer.

[3 marks]

c) A wireline AWGN channel of length 1000 km is used to transmit data via binary PAM. Regenerative repeaters are spaced 50 km apart along the system. Each segment of the channel has an ideal (constant) frequency response over the frequency band $0 \leq f \leq 1200$ and an attenuation of 1 dB/km.

i) What is the highest bit rate that can be transmitted without ISI?

ii) Determine the required $\frac{\mathcal{E}_b}{N_0}$ to achieve a bit error of $P_e = 10^{-7}$ for each repeater.

Hint: The probability of error for binary PAM transmission is $P_e = Q\left[\sqrt{\frac{2\mathcal{E}_b}{N_0}}\right]$

To find the Q-function values, see Table 1 on page 6.

[3 marks]

Q4 a) For an AWGN system, Maximum Likelihood detector criterion can be expressed as

$$\hat{S}_m = \max_m f(\boldsymbol{r}|s_m)$$

where $f(\boldsymbol{r}|s_m)$ is the conditional PDF of the observed vector $\boldsymbol{r}$ given s_m and

$$f(\boldsymbol{r}|s_m) = \prod_{k=1}^{N} \frac{1}{\sqrt{\pi N_0}} exp\left(-\frac{(r_k - s_{mk})^2}{N_0}\right).$$

By taking the natural logarithm of the likelihood function or any other appropriate method, show that the optimal detection can be obtained as

$$\hat{S}_m = \min_m \sum_{k=1}^{N} (r_k - s_{mk})^2.$$

You may refer to Figure Q4(a) for information regarding notations and symbols.

[5 marks]

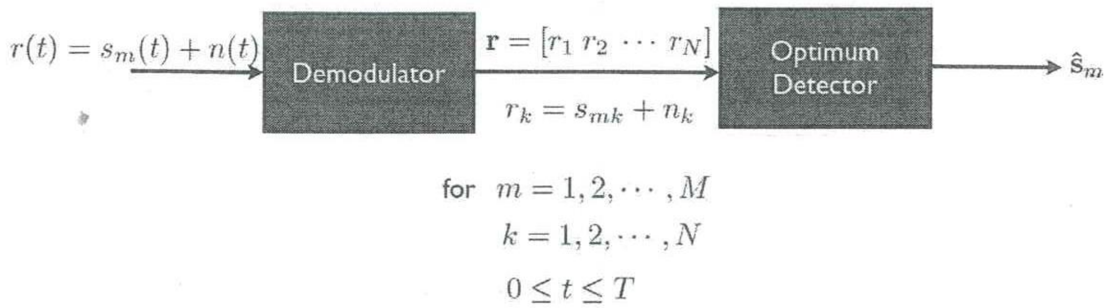

Figure Q4(a)

b) Analyze the Costas loop illustrated in Figure Q4(b) with respect to an input signal $x(t) = \sqrt{2}Aa(t)\cos(\omega t + \theta)$ and explain how this Costas loop is capable of simultaneously achieving carrier recovery and signal demodulation.

[5 marks]

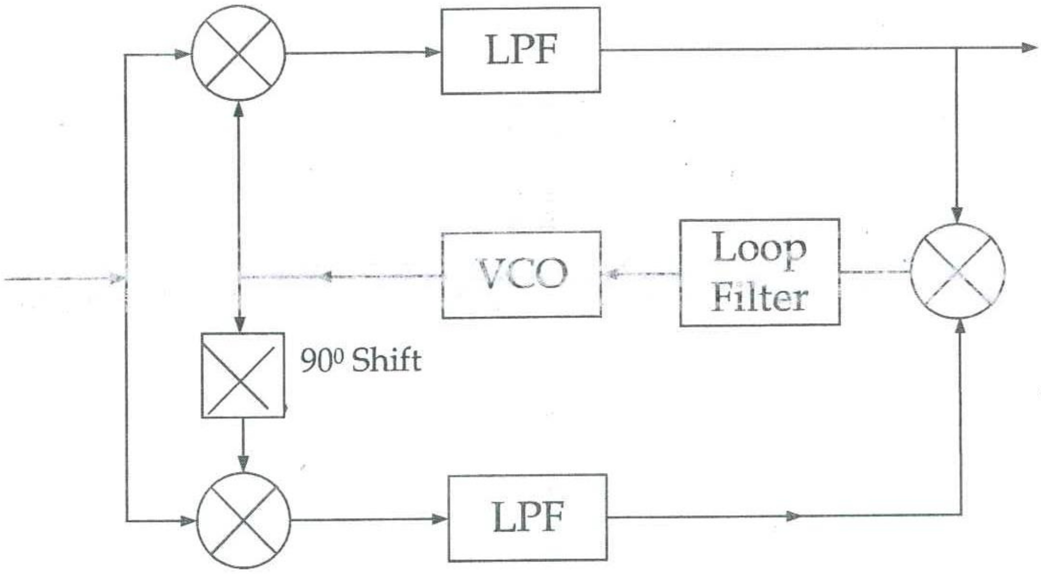

Figure Q4(b)

Q5 **a)** When designing a communication system, the selection of a modulation scheme is needed to meet the system requirements. Assume you have the option to select from PAM, PSK, FSK and QAM modulations. For each of the following systems mention your preferred modulation scheme and justify your selection.

i) A system where we will encounter large amplitude distortion in the channel with limited bandwidth.

ii) A data communication system, in which we have been allocated a certain bandwidth and want to obtain a high data rate.

iii) A system that supports simple modulation and wants to obtain a medium data rate.

[5 marks]

b) i) Discuss whether the detectors based on the Maximum A Posteriori (MAP) criterion and the Maximum Likelihood (ML) criterion would reach the same decision in a communication system.

ii) Consider the case of a binary PAM signal in which the two **equiprobable** signal points are $s_1 = -s_2 = \sqrt{\mathcal{E}_b}$ where $\mathcal{E}_b$ is the energy per bit, and the received signal vector is $r = \pm\sqrt{\mathcal{E}_b} + y_n(t)$. Here $y_n(t)$ is a zero-mean Gaussian random variable with variance $\sigma_n^2 = \frac{1}{2}N_0$. Determine the decision rule for the optimum MAP detector when the transmitted signal is corrupted with AWGN.

Hint: Conditional PDF for signal s_1 can be obtained as

$$f(r|s_1) = \frac{1}{\sqrt{2\pi}\sigma_n} exp\left(-\frac{\left(r-\sqrt{E_b}\right)^2}{2\sigma_n^2}\right).$$

[5 marks]

Table 1: Q-Function for $2 \leq x \leq 5.89$

x	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
2.00	2.2750e-02	2.2216e-02	2.1692e-02	2.1178e-02	2.0675e-02	2.0182e-02	1.9699e-02	1.9226e-02	1.8763e-02	1.8309e-02
2.10	1.7864e-02	1.7429e-02	1.7003e-02	1.6586e-02	1.6177e-02	1.5778e-02	1.5386e-02	1.5003e-02	1.4629e-02	1.4262e-02
2.20	1.3903e-02	1.3553e-02	1.3209e-02	1.2874e-02	1.2545e-02	1.2224e-02	1.1911e-02	1.1604e-02	1.1304e-02	1.1011e-02
2.30	1.0724e-02	1.0444e-02	1.0170e-02	9.9031e-03	9.6419e-03	9.3867e-03	9.1375e-03	8.8940e-03	8.6563e-03	8.4242e-03
2.40	8.1975e-03	7.9763e-03	7.7603e-03	7.5494e-03	7.3436e-03	7.1428e-03	6.9469e-03	6.7557e-03	6.5691e-03	6.3872e-03
2.50	6.2097e-03	6.0366e-03	5.8677e-03	5.7031e-03	5.5426e-03	5.3861e-03	5.2336e-03	5.0849e-03	4.9400e-03	4.7988e-03
2.60	4.6612e-03	4.5271e-03	4.3965e-03	4.2692e-03	4.1453e-03	4.0246e-03	3.9070e-03	3.7926e-03	3.6811e-03	3.5726e-03
2.70	3.4670e-03	3.3642e-03	3.2641e-03	3.1667e-03	3.0720e-03	2.9798e-03	2.8901e-03	2.8028e-03	2.7179e-03	2.6354e-03
2.80	2.5551e-03	2.4771e-03	2.4012e-03	2.3274e-03	2.2557e-03	2.1860e-03	2.1182e-03	2.0524e-03	1.9884e-03	1.9262e-03
2.90	1.8658e-03	1.8071e-03	1.7502e-03	1.6948e-03	1.6411e-03	1.5889e-03	1.5382e-03	1.4890e-03	1.4412e-03	1.3949e-03
3.00	1.3499e-03	1.3062e-03	1.2639e-03	1.2228e-03	1.1829e-03	1.1442e-03	1.1067e-03	1.0703e-03	1.0350e-03	1.0008e-03
3.10	9.6760e-04	9.3544e-04	9.0426e-04	8.7403e-04	8.4474e-04	8.1635e-04	7.8885e-04	7.6219e-04	7.3638e-04	7.1136e-04
3.20	6.8714e-04	6.6367e-04	6.4095e-04	6.1895e-04	5.9765e-04	5.7703e-04	5.5706e-04	5.3774e-04	5.1904e-04	5.0094e-04
3.30	4.8342e-04	4.6648e-04	4.5009e-04	4.3423e-04	4.1889e-04	4.0406e-04	3.8971e-04	3.7584e-04	3.6243e-04	3.4946e-04
3.40	3.3693e-04	3.2481e-04	3.1311e-04	3.0179e-04	2.9086e-04	2.8029e-04	2.7009e-04	2.6023e-04	2.5071e-04	2.4151e-04
3.50	2.3263e-04	2.2405e-04	2.1577e-04	2.0778e-04	2.0006e-04	1.9262e-04	1.8543e-04	1.7849e-04	1.7180e-04	1.6534e-04
3.60	1.5911e-04	1.5310e-04	1.4730e-04	1.4171e-04	1.3632e-04	1.3112e-04	1.2611e-04	1.2128e-04	1.1662e-04	1.1213e-04
3.70	1.0780e-04	1.0363e-04	9.9611e-05	9.5740e-05	9.2010e-05	8.8417e-05	8.4957e-05	8.1624e-05	7.8414e-05	7.5324e-05
3.80	7.2348e-05	6.9483e-05	6.6726e-05	6.4072e-05	6.1517e-05	5.9059e-05	5.6694e-05	5.4418e-05	5.2228e-05	5.0122e-05
3.90	4.8096e-05	4.6148e-05	4.4274e-05	4.2473e-05	4.0741e-05	3.9076e-05	3.7475e-05	3.5936e-05	3.4458e-05	3.3037e-05
4.00	3.1671e-05	3.0359e-05	2.9099e-05	2.7888e-05	2.6726e-05	2.5609e-05	2.4536e-05	2.3507e-05	2.2518e-05	2.1569e-05
4.10	2.0658e-05	1.9783e-05	1.8944e-05	1.8138e-05	1.7365e-05	1.6624e-05	1.5912e-05	1.5230e-05	1.4575e-05	1.3948e-05
4.20	1.3346e-05	1.2769e-05	1.2215e-05	1.1685e-05	1.1176e-05	1.0689e-05	1.0221e-05	9.7736e-06	9.3447e-06	8.9337e-06
4.30	8.5399e-06	8.1627e-06	7.8015e-06	7.4555e-06	7.1241e-06	6.8069e-06	6.5031e-06	6.2123e-06	5.9340e-06	5.6675e-06
4.40	5.4125e-06	5.1685e-06	4.9350e-06	4.7117e-06	4.4979e-06	4.2935e-06	4.0980e-06	3.9110e-06	3.7322e-06	3.5612e-06
4.50	3.3977e-06	3.2414e-06	3.0920e-06	2.9492e-06	2.8127e-06	2.6823e-06	2.5577e-06	2.4386e-06	2.3249e-06	2.2162e-06
4.60	2.1125e-06	2.0133e-06	1.9187e-06	1.8283e-06	1.7420e-06	1.6597e-06	1.5810e-06	1.5060e-06	1.4344e-06	1.3660e-06
4.70	1.3008e-06	1.2386e-06	1.1792e-06	1.1226e-06	1.0686e-06	1.0171e-06	9.6796e-07	9.2113e-07	8.7648e-07	8.3391e-07
4.80	7.9333e-07	7.5465e-07	7.1779e-07	6.8267e-07	6.4920e-07	6.1731e-07	5.8693e-07	5.5799e-07	5.3043e-07	5.0418e-07
4.90	4.7918e-07	4.5538e-07	4.3272e-07	4.1115e-07	3.9061e-07	3.7107e-07	3.5247e-07	3.3476e-07	3.1792e-07	3.0190e-07
5.00	2.8665e-07	2.7215e-07	2.5836e-07	2.4524e-07	2.3277e-07	2.2091e-07	2.0963e-07	1.9891e-07	1.8872e-07	1.7903e-07
5.10	1.6983e-07	1.6108e-07	1.5277e-07	1.4487e-07	1.3737e-07	1.3024e-07	1.2347e-07	1.1705e-07	1.1094e-07	1.0515e-07
5.20	9.9644e-08	9.4420e-08	8.9462e-08	8.4755e-08	8.0288e-08	7.6050e-08	7.2028e-08	6.8212e-08	6.4592e-08	6.1158e-08
5.30	5.7901e-08	5.4813e-08	5.1884e-08	4.9106e-08	4.6473e-08	4.3977e-08	4.1611e-08	3.9368e-08	3.7243e-08	3.5229e-08
5.40	3.3320e-08	3.1512e-08	2.9800e-08	2.8177e-08	2.6640e-08	2.5185e-08	2.3807e-08	2.2502e-08	2.1266e-08	2.0097e-08
5.50	1.8990e-08	1.7942e-08	1.6950e-08	1.6012e-08	1.5124e-08	1.4283e-08	1.3489e-08	1.2737e-08	1.2026e-08	1.1353e-08
5.60	1.0718e-08	1.0116e-08	9.5479e-09	9.0105e-09	8.5025e-09	8.0224e-09	7.5686e-09	7.1399e-09	6.7347e-09	6.3520e-09
5.70	5.9904e-09	5.6488e-09	5.3262e-09	5.0215e-09	4.7338e-09	4.4622e-09	4.2057e-09	3.9636e-09	3.7350e-09	3.5193e-09
5.80	3.3157e-09	3.1236e-09	2.9424e-09	2.7714e-09	2.6100e-09	2.4579e-09	2.3143e-09	2.1790e-09	2.0513e-09	1.9310e-09