UNIVERSITY OF RUHUNA

Faculty of Engineering

End-Semester 6 Examination in Engineering: September 2023

Module Number: IS6303 **Module Name: Mathematical Modelling**

[Three hours]

[Answer **all questions,** each question carries twelve marks]

Q1.

a) Name two problems that might be modelled mathematically. Why do you think mathematics may provide a key to solving each problem? What is the added value in each case?

[2 Marks]

b) If you jump out of an airplane at high altitude but do not open your parachute, you will soon fall at a constant velocity called "*terminal velocity*". Suppose that at $t = 0$, you jump and when $t = 15$s, your wrist altimeter shows that your distance from the ground, d, is 4000m. When $t = 40$s, you have dropped to $d = 2000$m. Assume that you are at your terminal velocity by the time $t = 15$s.

i.) Write the particular equation expressing d in terms of t.

ii.) If you neglect to open your parachute, when will you hit the ground?

iii.) According to your mathematical model, how high was the airplane when you jumped?

iv.) The plane was actually at 5000 meters when you jumped. How do you reconcile this fact with your answer in part (iii)?

[4 Marks]

c) i.) Briefly explain the importance of 'Dimensional Analysis' in mathematical modelling of real-world problems.

ii.) Clearly state the 'Buckingham Pi Theorem'.

iii.) The tip deflection (δ) of a cantilever beam is a function of tip load (W), beam length (l), second moment of area (I) and Young's modulus (E). Using dimensional analysis, find a relation between the tip deflection and the other parameter(s) of the problem.

[6 Marks]

Q2.

a) i.) State the advantages and disadvantages (2 each) of 'Linear Programming' used in mathematical modelling.

ii.) Briefly explain the steps of solving a linear programming problem using 'Graphical Method'.

[3 Marks]

b) A company manufactures two types of wooden toys; soldiers and trains. A soldier sells for Rs. 2,700 and uses Rs. 1,000 of raw materials. Each soldier that is manufactured costs Rs. 1,400 for labor and overhead. A train sells for Rs. 2,100 and uses Rs. 900 of raw materials. Each train that is manufactured costs Rs. 1,000 for labor and overhead. The soldiers and trains require two types of skilled labor; carpentry and finishing. A soldier requires 2 hours of finishing labor and 1 hour of carpentry labor. A train requires 1 hour of finishing and 1 hour of carpentry labor. Each week, the company can obtain all the needed raw material, but only 100 finishing hours and 80 carpentry hours are available. Demand for trains is unlimited, but at most 40 soldiers are bought each week. The company wants to maximize the weekly profit.

i.) State the objective function and the problem constraints.

ii.) Using the graphical method, find how many of each toy should be made to maximize the weekly profit.

[9 Marks]

Q3.

a) Briefly explain about 'Balanced and Unbalanced Transportation Problems' and the use of 'Dummy demand point and Dummy supply point' in transportation problems.

[2 Marks]

b) Briefly explain the procedures of ' The Northwest Corner Method' and 'Stepping-Stone Solution Method' used to solve transportation problems.

[2 Marks]

c) Oranges are grown, picked, and then stored in warehouses in Bibile, Badulla and Monaragala. These warehouses supply oranges to markets in Ampara, Ratnapura, Kandy and Galle. The following table shows the supply costs per truckload (Rs.1000s), supply and demand.

From	To				Supply
	Ampara	Ratnapura	Kandy	Galle	
Bibile	9	14	12	17	220
Badulla	11	10	6	10	200
Monaragala	12	8	15	7	100
Demand	150	170	100	150	

i.) Set up the transportation tableau for this problem and determine the initial solution using the Northwest corner method.

ii.) Find the total cost for the initial feasible solution.

iii.) Solve the problem using the Stepping-Stone Solution Method.

iv.) Are there multiple optimal solutions? If so, explain and identify them.

[6 Marks]

d) Suppose in part (c), the supplies are prohibited from Bibile to Kandy due to ongoing road construction works. What modification to the transportation tableau would you do to consider this prohibited root? Without repeating the calculations, briefly explain any specific points of solving the above problem with this modification.

[2 Marks]

Q4.

a) Briefly explain what is meant by,

i.) 'Row reductions' and 'Column reductions' performed in assignment problems.

ii.) Prohibited Assignment.

[2 Marks]

b) Give examples (2 each) for maximization and minimization assignment problems occurring in real world with related to engineering/management.

[2 Marks]

c) A Manager of a Transportation company needs to order 5 trucks out of a fleet to be present at 5 specific locations for loading goods that are awaiting shipment in 7 trucks at different locations. The transportation costs (Rs. 1000s) for each truck to different loading locations are shown in the table below.

Trucks	Loading Locations				
	A	B	C	D	E
1	30	29	28	29	21
2	25	31	29	30	20
3	18	19	30	19	18
4	32	18	19	24	17
5	27	21	22	25	16
6	19	20	19	19	14
7	22	30	23	18	16

Find the assignment of trucks to minimize the total transportation cost by using 'Hungarian' method. If there are multiple optimal solutions, identify them and calculate the total transportation cost.

[8 Marks]

Q5.

a) i.) Briefly explain what is meant by 'a difference equation'.

ii.) Briefly explain how you can identify the linear stability of a fixed point.

[2 Marks]

b) Assume that,

- there are 2 chemicals in the body, U and V.
- the body filters out 10% of U and 15% of V each day through the kidneys.
- liver enzymes metabolize 30% of U into V and 20% of V into U each day.

In addition, we assume that, each day x mg of V is taken in but no U is taken in.

i.) What should x be so that the equilibrium value for V is 100 mg?

ii.) What will be the resulting equilibrium value for U given the prescribed dosage of V found in part (i)?

[5 Marks]

b) To buy a boat in 2 years, Mr. Perera's family plans to save Rs. 25,000 a month in an account that pays 12% interest, compounded monthly.

i.) Find the total amount of the payments.

ii.) Find the value of the account in 2 years.

iii.) Find the single deposit in that account that would give the same future value.

[5 Marks]