UNIVERSITY OF RUHUNA

Faculty of Engineering

End-Semester 1 Examination in Engineering: December 2023 (Repeat)

Module Number: ME1202 Module Name: Introduction to Mechanical Engineering

[Three Hours]

[This paper consists of 5 questions in 5 pages. Answer **all questions**, each question carries **12** marks]

Clearly state all the assumptions that you may make.
To get full marks, make sure that you have answered with correct SI units and standard notations.
Take gravitational acceleration, $g = 9.8\ m/s^2$

Q1. a) The position of a point P in space (Figure Q1(a)) is described by the position vector, $\boldsymbol{r} = \underline{r}(t)$. The distance measured along the path is s.

i) Define normal and tangential coordinate system to represent the particle motion using unit vectors along normal direction ($\underline{e}_n$) and tangential direction ($\underline{e}_t$).

ii) Obtain expressions for the speed, the velocity, and the acceleration of the particle using above unit vectors.

[5 Marks]

b) Figure Q1(b) shows a guided slot in horizontal plane and a pin that can move inside. The y coordinate of the path of the pin is given as $(x - kx^2)\ m$. The pin acceleration is given by $\ddot{x} = (1 + 0.2t)\ m/s^2$ and the initial conditions of the pin's motion are $x(0) = 0.6\ m$, $\dot{x}(0) = 0.6\ m/s$. Calculate the followings when the $t = 0.5\ s$. Given that $k = 1\ m^{-1}$.

i) Speed and the acceleration of the pin.

ii) Radius of the curvature.

[7 Marks]

Q2 a) Consider a mass-spring system (Figure Q2(a)) that consists of a block (mass, m) attached to a wall with a spring of constant k. The kinematic friction coefficient between the block surface and the horizontal plane is given by μ_k. A horizontal force, $F(t)$ is applied parallel to the plane. Taking the friction force on the cube as F_f and the displacement of the block as $x(t)$,

i) Draw two free body diagrams showing the forces on the block
- when it is having a rectilinear motion in the same direction as of F and
- exactly opposite direction to F.

ii) Obtain the equation of the motion for the block for each motion.

[5 Marks]

(Q2 continuous to next page)

b) Figure Q2(b) shows a plane inclined to the horizontal plane by $\alpha°$. There are two paths created as slots on the plane as straight path and a semi-circular path with radius R. Two equal masses with M each are released at the bottom of the paths with an initial speed V.

i) Show that both masses reach to the top of the path with same speeds if paths are smooth and calculate the speed.

ii) If the static and kinetic friction coefficients of the bottom surface of the path are μ, obtain initial conditions for the velocity that should be satisfied by each mass to reach the top.

iii) Determine which is the slowest path.

[7 Marks]

Q3 a) States the following theories used to analyse the dynamics of system of particles. Define each term and give their mathematical expressions.

i) Conservation of linear momentum.

ii) Conservation of energy.

[5 Marks]

b) A motor with a pulley system is shown in Figure Q3(b). The motor exerts a force F on the crate(B) with mass of 40 kg given by the function,

$$F(t) = \begin{cases} 150 + 50t, & t < 6 \\ 450, & t \geq 6 \end{cases}$$

The maximum power of the motor is 5 kW and the motor pulley radius is 5 cm. Given that the crate moving downward at 10 m/s when $t = 0$. Determine,

i) the time at which the crate changes its direction of motion.

ii) the speed and direction of the crate at $t = 6\ s$.

[7 Marks]

Q4 a) i) State the Newton's second law of motion for moments for system of particles.

ii) Hence, derive the principle of angular impulse and momentum for system of particles.

[5 Marks]

b) The 8 kg carriage and ball assembly (four balls, each of the four balls weight 1.2 kg) shown in Figure Q4(b) consist of horizontal guides, two assemblies of balls (one at the front and one at the back) with light rods which rotates about a shaft at O. Length of the front rod is 0.9 m and length of the back rod is 0.6 m. The carriage moves horizontally in its guide with a speed of 1.2 m/s. The assembly on the front face rotates counterclockwise at a speed of 80 rev/min, and the assembly on the back side rotates clockwise at a speed of 100 rev/min. Calculate,

i) the velocities of four balls with respect to O

ii) the kinetic energy of the entire system

iii) magnitude of the total linear momentum

iv) magnitude of the total angular momentum

[7 Marks]

Q5 a) Briefly describe 3 main types of rigid body planner motions using suitable sketches.

[3 Marks]

b) List 3 mechanisms (except the Gear box mechanism) used in an automobile with their motion conversion.

[3 Marks]

c) The system shown in Figure Q5(c) consists of two gears A (radius 75 mm) and B (radius 225 mm). A cylinder is attached to one end of the cord which is wrapped around pulley D (radius 125 mm). D is rigidly attached to gear B so that both rotates at the same speed. The motor attached to gear A turns it with angular acceleration of $\alpha_A = (t^2 + 1)\ rad/s^2$ clockwise. If the gear A starts from rest, determine the following after 5 $seconds$,

i) the velocity of the cylinder

ii) the distance that the cylinder travels

[6 Marks]

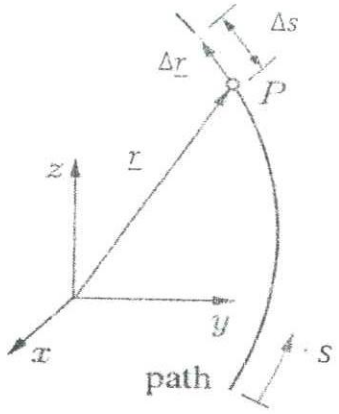

Figure Q1(a)

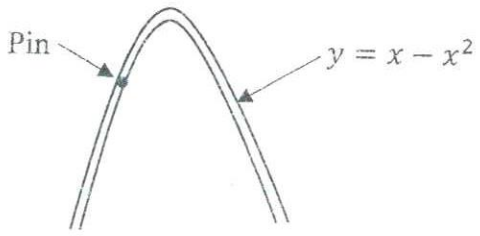

Figure Q1(b)

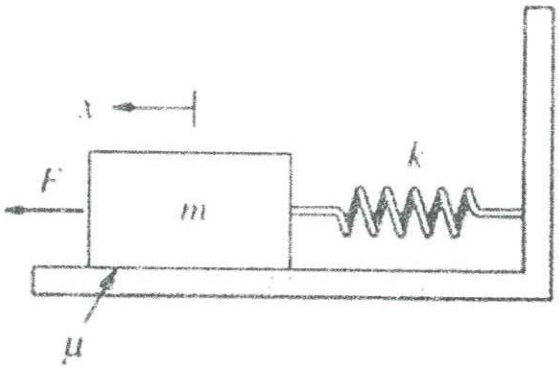

Figure Q2(a)

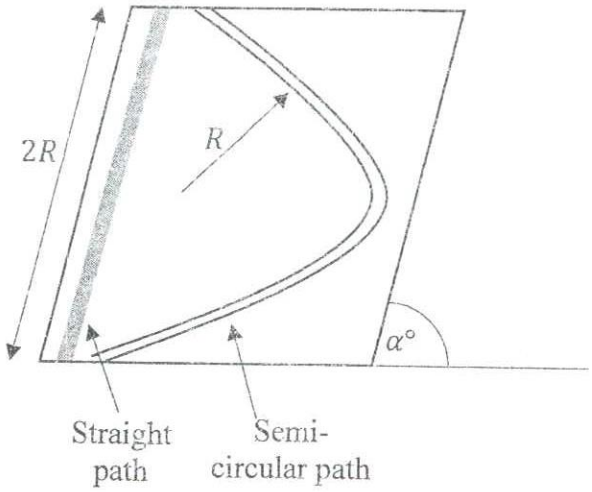

Figure Q2(b)

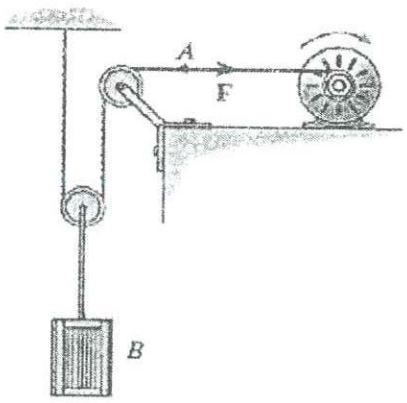

Figure Q3(b)

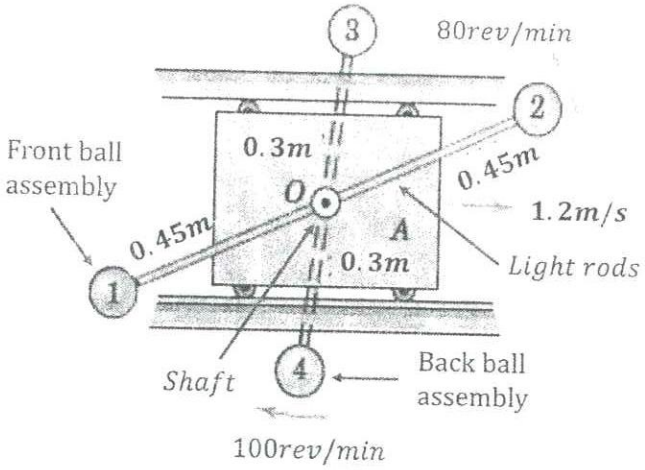

Figure Q4(b)

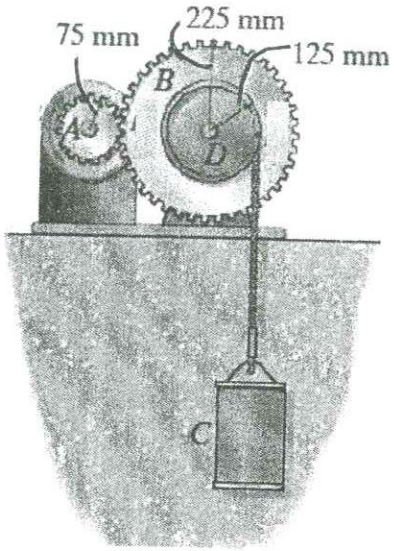

Figure Q5(c)