University of Ruhuna

Bachelor of Science General Degree
Level I (semester I) Examination – June 2015

Subject: Mathematics
Course Unit: AMT112β/MMA1113
(Mathematical Foundation of Computer Science)
Time: Two (02) hours

Answer 04 questions only

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(1) (a) Prove that, if n is an integer and 3n+4 is even, then n is even using

(i) direct proof
(ii) contrapositive proof
(iii) proof by contradiction.

(b) State the Generalized Pigeonhole principle.
(i) 5 computers on a network are connected to at least 1 other computer. Show there are at least two computers that are have the same number of connections.

(ii) Consider 5 distinct points (x_i, y_i) with integer values, where i= 1,2,3,4,5. Show that the midpoint of at least one pair of these five points also has integer coordinates.

(c) State the inverse, converse and the contrapositive of the statement:
"If I do well in this examination, then I will have a good average".

(d) Symbolize and test the following argument for validity by using a **truth table**.

If you send me a message, then I will come to the welcome party. If you do not send me a message then I will go to see a film. Therefore I will come to the welcome party or I will go to see a film.

(2) (a) Explain what is meant by saying an argument is valid.

Test for the validity of the following argument using **pattern proof**:

If I join the picnic I can get a better knowledge about the wild-life. If I don't join the picnic I will go for fishing. If I get a better knowledge about the wild life, I will write a book about wild-animals. Therefore if I do not go for fishing, I will write a book about wild-animals.

(b) Consider the following sentences:

All students are sporty.
Anyone who is sporty and determined will perform well in the tournament.
Anyone who performs well will win in his/her event.
Sugath is a determined student.

(i) Represent the above facts in axiom forms.
(ii) Translate the above axioms into clausal form.
(iii) Use these clausal forms to prove that "Sugath will win his event.".

(3) (a) Explain what is meant by a tautology and a contradiction.
Using the truth tables, check whether the followings are tautologies, contradictions or neither:
(i) $(P \wedge P \rightarrow Q) \rightarrow Q$,
(ii) $[(P \vee Q) \wedge (\sim Q \vee R)] \rightarrow \sim P$

(b) A computer has a word length of 8 bits and uses two's complement method for calculations. Translate -73 into the number format used by the above computer.

(c) Explain the method of simplifying 49 – 59 using two's complements in an 8-bit word-length computer.

(d) Explain the method of performing the calculation $(107_{10})/(17_{10})$ in a computer with 8-bit word-length that uses two's complement.

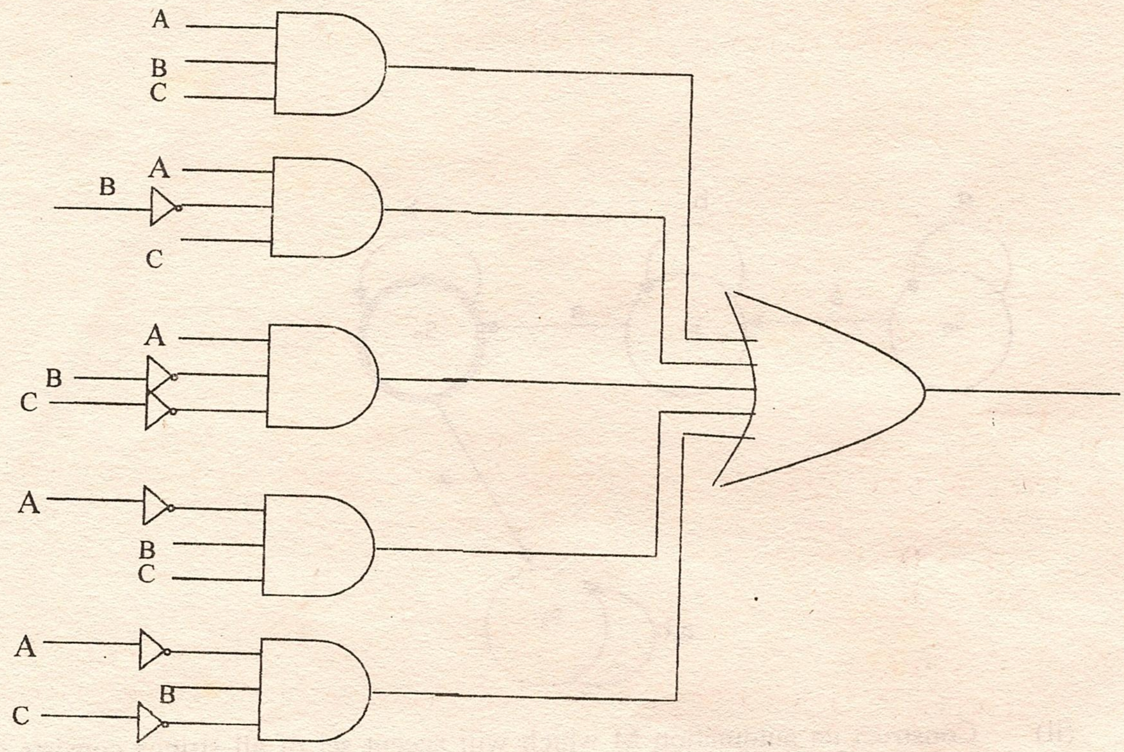
(4) (a) (i) If the fundamental product P_1 is contained in another fundamental product P_2 then show that
$$P_1 + P_2 = P_1.$$

(ii) Define what is meant by
(α) a sum-of-product expression
(β) a complete-sum-of-product expression.

(iii) Show that the sum-of-product form of the expression
$A.B'.C + A.B.C + A.[B'+C]' + A'.B.C + [A+B'+C]'$ is equivalent to $A.C+B$.

(iv) Hence find the complete-sum-of-product form of the expression.

(b) (i) Write down the Boolean expression for the following logic circuit.



(ii) Use a Karnaugh map to find a minimal sum-of-products expression for the network.

(iii) Sketch the logic circuit for the minimal sum-of product expression you obtained in the part (ii) above.

(5)(a) (i) Define the dual of a proposition concerning a Boolean Algebra $\mathfrak{B}$.
Write the dual of the Boolean expression

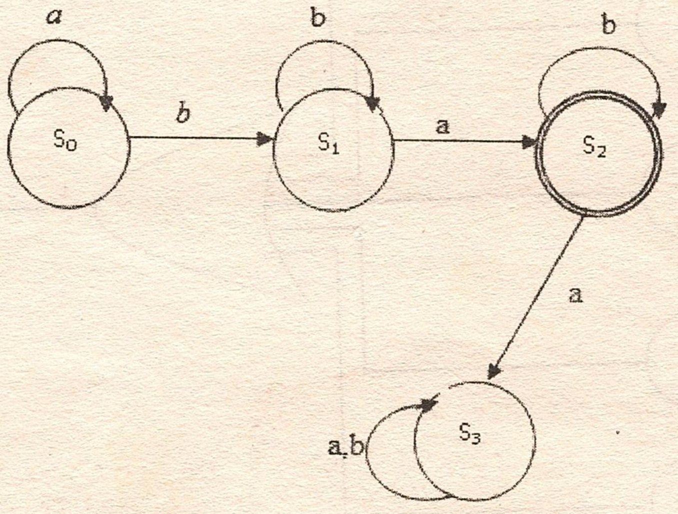
$$(A.\overline{B}) + 1 = 1.$$

(ii) For all A, B in $\mathfrak{B}(.,+,\bar{\ },0,1)$, prove that $A + B = \overline{(\overline{A}.\overline{B})}$

(iii) Find the complete sum of product form of the expression

$$E = (xy + z')[xz' + yz]$$

(b) (i) Describe the sets recognized by the following finite state machine:



(ii) Construct an automation M which will accept set of all strings consists of one or more 1's followed by a 0.

(6). (i) Use the expand, guess and verify method to show that the closed-form-solution of the recurrence relation

$$S(n) = 4 * S(n-1) \quad \text{for } n \geq 2$$

with the base value S(1) = 2 is given by

$$S(n) = 2^{2n-1}$$

(ii) **Show that** the solution to the linear first-order recurrence relation of the form

$$S(n) = S(n-1) + g(n)$$

with the base value S(1) is given by

$$S(n) = S(1) + \sum_{i=2}^{n} g(i).$$

The first four numbers of a sequence are given by 5, 14, 26 and 41.

Find a recurrence formula of the form S(n) = S(n-1) + g(n) for the n^{th} number in the sequence.
Hence find a formula for the n^{th} number in the sequence.

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