University of Ruhuna

B.Sc.(General) Degree - Level I

(Semester I) Examination - June/July 2015

Subject: Industrial Mathematics/Applied Mathematics
Course Unit: IMT111β/AMT111β/MAM1133
(Classical Machanics I)

Time: Two (02) Hours

Answer four(04) questions only

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1. (a) A particle of mass m moves in two dimensional space so that its position vector at time t is $\vec{r}(t) = 4\cos\frac{t}{2}\,\underline{\mathrm{i}} + 4\sin\frac{t}{2}\,\underline{\mathrm{j}}$.

(i) Find the velocity and the acceleration when $t = \pi$ and $t = \frac{3\pi}{2}$.

(ii) Determine an equation in x and y (independent of t) for the path of the particle and sketch in $xy-$plane.

(b) A car of mass m breaks to stop by a combination of Coulomb's and Stoke's frictional damping (resistance forces), so that the total resistance is $-\mu mg - k\dot{x}$, $\dot{x} > 0$, where x is the distance from the point at which the brakes are applied, and $\mu > 0$, $k \geq 0$ are constants. Show that

(i) the car breaking at the speed v_0 at time $t = 0$, will come to the rest at a time $T_s = \frac{v_0}{\mu g}$ when $k = 0$ (i.e. Pure Coulomb's damping) and

(ii) the stopping time T for full damping (i.e. $\mu > 0$ and $k \neq 0$) with same initial velocity is given by

$$T = \frac{m}{k}\log_e\left(1 + \frac{k}{m}T_s\right).$$

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2. a) In the usual notation, show that the velocity and acceleration components of a moving particle are given by

(i) $\underline{v} = \dot{r}\hat{r} + r\dot{\theta}\underline{\hat{\theta}} + \dot{z}\hat{z}$

(ii) $\underline{a} = (\ddot{r} - r\dot{\theta}^2)\hat{r} + \frac{1}{r}\frac{d}{dt}(r^2\dot{\theta})\underline{\hat{\theta}} + \ddot{z}\hat{z}$

in cylindrical polar coordinates.

Continued.

b) A particle of mass m is projected horizontally at the point $z = -a/2$ with velocity v_0 along the inner surface of a frictionless hollow sphere of radius a given by $r^2 + z^2 = a^2$ in cylindrical polar coordinates, where z is measured upwards from the centre of the sphere.

(i) Applying Newton's second law for the motion show that $r^2\dot{\theta} = h$, where h is a constant.

(ii) Using the conservation of energy of the system show that

$$\frac{m}{2}\left(\frac{a^2\dot{z}^2}{a^2 - z^2} + \frac{h^2}{a^2 - z^2}\right) + mgz = constant.$$

(iii) Obtain an equation of z for finding the minimum and maximum height that the particle reaches during the ensuing motion.

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3. a) Consider a system of masses m_i, $i = 1, 2, \ldots, n$ whose position vectors are r_i, $i = 1, 2, \ldots, n$ respectively.

(i) Define the total angular momentum $\underline{\mathbf{H}}_0$ about the origin and the total kinetic energy T.

(ii) In the usual notation show that $\frac{d\mathbf{H}_0}{dt} = \sum_{i=1}^{n} \underline{\mathbf{r}}_i \wedge \underline{\mathbf{F}}_i$ and $T = T_G + \frac{1}{2}MV_G^2$.

b) If a rigid body rotates about a fixed axis with angular velocity $\underline{\omega}$ and its moment of inertia about the fixed axis is I, write down the expressions for the angular momentum and the kinetic energy of the rotation.

c) (i) A cylinder of radius a and radius of gyration K rolls down without slipping on an inclined plane of angle α and length l, starting from the top of the inclined plane.

Show that when it reaches the bottom of the inclined plane the speed v_1 will be $\sqrt{\frac{2gla^2 \sin\alpha}{(a^2 + K^2)}}$.

(ii) Consider the similar system as given in part c(i) above but replacing the cylinder by a hollow cylinder of same radius and mass. Deduce that the velocity v_2 at the bottom of the inclined plane takes the relation $v_1 : v_2 = 2 : \sqrt{3}$.

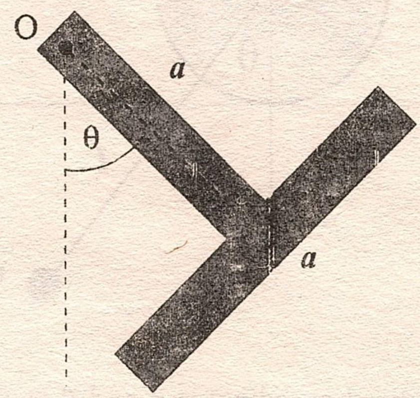
Continued.

4. a) Consider a uniform rod of mass m and length a.

(i) Show that the moment of inertia about the axis which goes through an end and perpendicul to the rod is $\frac{ma^2}{3}$.

(ii) Using the parallel axis theorem show that the moment of inertia about the axis which goes through the mid point (centre of mass) and perpendicular to the rod is $\frac{ma^2}{12}$.

b) Two rods each of length a and mass m are joined together at a right angle to form a T−square. This T−Square is then hung on a nail on which it is free to rotate in its vertical plane about O as shown in the figure.



(i) Show that the moment of inertia of T-square about O is $\frac{17ma^2}{12}$. (You may use the parallel axis theorem).

(ii) Find the period of small oscillations of the T−square.

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5. a) In the usual notation, obtain the Euler's equations for the motion of a rigid body with one point fixed.

b) Consider the motion of a top (a rigid body motion with one point fixed) with principal moments of inertia I_1, I_2 and I_3 such that $I_1 = I_2$ and $I_1 < I_3$.

(i) Write down the Euler's equations of motion for free torque motion with one point fixed.

(ii) If the initial angular velocity is $\underline{\omega_0} = (\Omega_1, \Omega_2, \Omega_3)$ solve the system for angular velocity $\underline{\omega}$ at any time.

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6. a) In the usual notation, the Lagrange's equations for a dynamical system is given by:

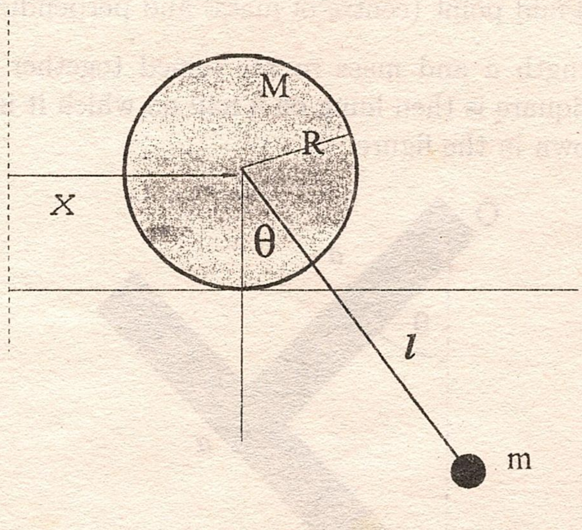
$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_j}\right) - \frac{\partial T}{\partial q_j} = Q_j; \qquad j = 1, 2, ..., n.$$

Deduce the Lagrange's equations for a holonomic conservative dynamical system of the form:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_j}\right) - \frac{\partial L}{\partial q_j} = 0; \qquad j = 1, 2, ..., n.$$

Continued.

b) A uniform circular disk of mass M and radius R is free to move without slipping on a flat horizontal surface. A pendulum is suspended from a frictionless pivot mounded at the axis of the disk as shown in the figure. The pendulum consists of a light rod of length l with a point mass m attached to its end.



(i) Considering the generalized coordinates x and θ as shown in the figure, show that the Lagrange function L is given by

$$L = \frac{3}{4}M\dot{x}^2 + \frac{m}{2}\left(\dot{x}^2 + l^2\dot{\theta}^2 + 2l\dot{x}\dot{\theta}\cos\theta\right) + mgl\cos\theta.$$

(ii) Use Lagrange's equation to obtain the equations of motion.

c) Consider the given system in part (b) above with $M = m$ and small θ.

(i) Obtain the expressions for generalized (canonical) momentums P_θ and P_x.

(ii) Explain how you would obtain the Hamiltonian function H in terms of generalized coordinates θ and x and generalized momentums P_θ and P_x (It is not required to simplify the expression).

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