University of Ruhuna

Bachelor of Science General Degree Level I
(Semester I Examination)

July 2016

Subject: Applied Mathematics/Industrial Mathematics
Course Unit: AMT111β/IMT111β(Classical Mechanics-I)

Time: Two (02) Hours

Answer 04 Questions only.

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1. (a) A particle P moves along the x-axis with its acceleration a at time t given by $a = (6t - 4)ms^{-2}$.
Initially P is at the point $x = 20m$ and is moving with speed $15ms^{-1}$ in the negative x-direction. Find the velocity and displacement of P at time t. Also, find the time at which P comes to rest and its displacement at this time.

(b) A particle P of mass m moves along a straight line through a point O and at any instant the distance OP is x. When $x > a$, the particle is attracted towards O by a force $\frac{mk}{x^2}$, and when $x < a$ the particle is repelled from O by a force $\frac{mka}{x^3}$, where k is a constant. If the particle is released from rest at a distance $2a$ from O show that it will come to rest instantaneously when $x = \frac{a}{\sqrt{2}}$.

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2. a) Show in the usual notation, that the velocity and acceleration components of a moving particle are given in plane polar coordinates by:

(i) $\underline{\mathbf{v}} = \dot{r}\underline{\hat{\mathbf{r}}} + r\dot{\theta}\underline{\hat{\theta}}$

(ii) $\underline{\mathbf{a}} = (\ddot{r} - r\dot{\theta}^2)\underline{\hat{\mathbf{r}}} + \frac{1}{r}\frac{d}{dt}(r^2\dot{\theta})\underline{\hat{\theta}}$.

b) A particle A of mass m moves on a smooth horizontal table and is connected by a light inextensible string passing through a smooth hole O in the table to a particle B of the same mass m, which moves along the vertical line through O. Initially B is at rest and A is distance a from O, moving with velocity $\sqrt{\frac{ga}{3}}$ perpendicular to OA. Show, in the subsequent motion that the distance $r = OA$ lies between a and $\frac{a}{2}$. Show also that the tension of the string is

$$\frac{1}{6}mg\left(3 + \frac{a^3}{r^3}\right).$$

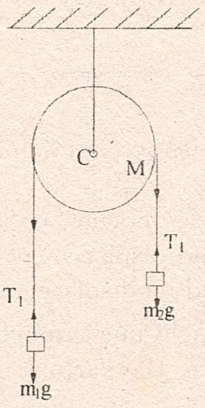
Continued.

3. a) For a system of particles, define the total angular momentum $\underline{\mathbf{H}}_0$ about the origin and show, in the usual notation, that

(i) $\dfrac{d\underline{\mathbf{H}}_0}{dt} = \sum_{i=1}^{n} \underline{\mathbf{r}}_i \wedge \underline{\mathbf{F}}_i$ and

(ii) $\underline{\mathbf{H}}_0 = \underline{\mathbf{r}}_G \wedge M\underline{\mathbf{V}}_G + \underline{\mathbf{H}}_G$.

b) Two masses m_1 and m_2 are connected by an inextensible string of negligible mass which passes over a frictionless pulley of mass M, radius of gyration K which can rotate about a horizontal axis through C perpendicular to the pulley (see the following figure).



Find the

(i) accelerations of m_1 and m_2 and

(ii) tensions T_1 and T_2 of the string.

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4. a) Suppose that a rigid body is moving with one point fixed. In the usual notation, the angular momentum of the rigid body about the instantaneous axis through the fixed point is given by $\underline{\mathbf{H}} = (H_1, H_2, H_3) = \sum m_i(\underline{\mathbf{r}}_i \wedge (\underline{\omega} \wedge \underline{\mathbf{r}}_i))$. Show, in the usual notation, that

$$H_1 = I_{xx}\omega_1 + I_{xy}\omega_2 + I_{xz}\omega_3$$
$$H_2 = I_{xy}\omega_1 + I_{yy}\omega_2 + I_{yz}\omega_3$$
$$H_3 = I_{xz}\omega_1 + I_{yz}\omega_2 + I_{zz}\omega_3.$$

Use the above result to show that the principal moments of inertia I are given by

$$\begin{vmatrix} I_{xx} - I & I_{xy} & I_{xz} \\ I_{xy} & I_{yy} - I & I_{yz} \\ I_{xz} & I_{yz} & I_{zz} - I \end{vmatrix} = 0.$$

Continued.

b) Let G be the center of gravity of a uniform solid rectangular parallelepiped with sides $2a$, $2a$, a. Find

(i) the moments of inertia and

(ii) the products of inertia

of the parallelepiped about a system of rectangular axes through G and parallel to the sides of the parallelepiped.

Also, find the principal moments of inertia and the directions of principal axes of the parallelepiped.

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5. a) Obtain in the usual notation, the Euler's equations for the motion of a rigid body with one point fixed.

b) A rigid body is free to rotate about its center of gravity, G, without external forces. The principal moments of inertia about G are $6I$, $3I$ and I units respectively. Initially the body is given an angular velocity $\underline{\omega}_0 = (k, 0, 3k)$. Here I and k are constants. Show that after time t the angular velocity $\underline{\omega} = (\omega_1, \omega_2, \omega_3)$ satisfies the equations

$$5\omega_1^2 + \omega_2^2 = 5k^2$$
$$9\omega_2^2 + 5\omega_3^2 = 45k^2$$
$$\omega_2 = -\sqrt{5}\tanh(\sqrt{5}kt).$$

Also, find ω_1 and ω_3.

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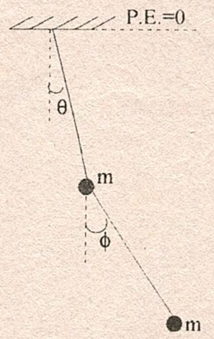
6. a) The Lagrange's equations for a dynamical system is given in the usual notation by:

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_j}\right) - \frac{\partial T}{\partial q_j} = Q_j; \qquad j = 1, 2, ..., n.$$

Deduce the Lagrange's equations for a holonomic conservative dynamical system of the form:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_j}\right) - \frac{\partial L}{\partial q_j} = 0; \qquad j = 1, 2, ..., n.$$

b) A double pendulum consists of two simple pendulums of same length l and same mass m with the cord of one pendulum attached to the bob of the other pendulum whose cord is fixed to pivot (see following figure).



Continued.

Show that the kinetic energy, T, and the potential energy, V, of the system are given by:

(i) $T = \frac{1}{2}ml^2\left(2\dot{\theta}^2 + \dot{\phi}^2 + 2\dot{\theta}\dot{\phi}\cos(\phi - \theta)\right)$ and

(ii) $V = -mgl(2\cos\theta + \cos\phi)$,

respectively.

Determine the equations of motion for small oscillations using Lagrange's equations.
