University of Ruhuna

Bachelor of Science Special Degree
Level I (Semester II) Examination - 2016

Subject: Mathematics
Course Unit: MSP324α(Complex Analysis) . **Time: One (01) Hour**

Answer All Questions

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1. a) What is meant by the Cauchy principal value of improper real integral of the form

$$I = \int_{-\infty}^{\infty} f(x)dx.$$

b) Evaluate the following real integral:

$$\int_0^{\infty} \frac{x \sin x}{(x^2+2^2)^2} dx,$$

by stating clearly the results you may use in the calculation.

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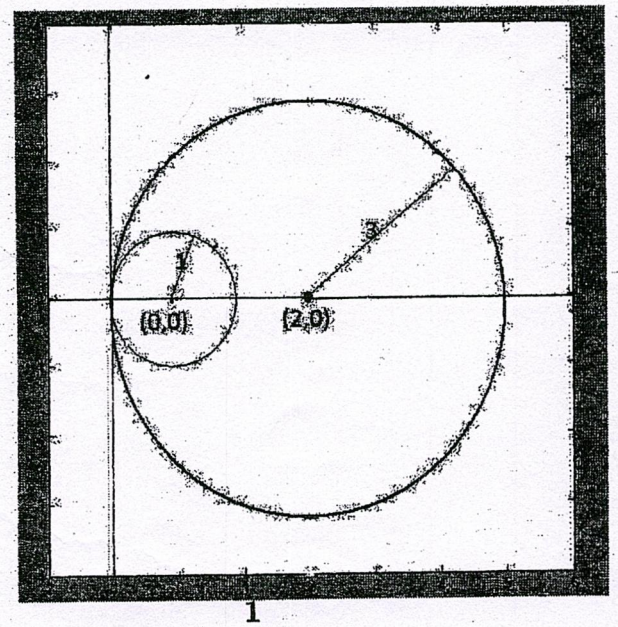
2. a) Define in integral form, the winding number of a closed curve γ around a point z_0 which does not lie on the curve γ.

b) State clearly the second version of the residue theorem.

c) Evaluate the integral $\int_C f(z)dz$ where

$$f(z) = \frac{z+1}{z(z-2)(z-2i)},$$

and C is a closed curve starting at $z = -1$ such that $C = C_1 + C_2$ and C_1 and C_2 are closed curves as shown in following figure.



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3. a) Define each of the following terms:

(i) meromorphic function,

(ii) holomorphic function,

(iii) logarithmic derivative of a function

b) Let f be a meromorphic function on domain D and it has a pole at z_0 with order m. Prove that the logarithmic derivative has a simple pole at z_0 with residue $-m$.

c) State clearly the Rouché's theorem

Find the number of zeros of the polynomial

$$F(z) = z^9 - 4z^6 - z + 1$$

inside the circle $|z| = 1$.