University of Ruhuna

B.Sc.(General) Degree

Level II (Semester I) Examination - September 2017

Subject: Industrial Mathematics/Applied Mathematics
Course Unit: AMT211β/IMT211β(Fluid Dynamics) **Time: Two (02) Hours**

Answer Four (04) Questions only.

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1. a) In the usual notation, obtain the equation of continuity in the form $\dfrac{\partial \rho}{\partial t} + \underline{\nabla}\cdot(\rho \underline{q}) = 0$ for a motion of a fluid.
If the fluid is incompressible, then deduce that $div\ \underline{q} = 0$.

   b) Show that the motion given by

$$\underline{q} = \left(\frac{by - ax}{x^2 + y^2}, -\left(\frac{ay + bx}{x^2 + y^2}\right), 0 \right) \text{ ;where } a, b \text{ are constant,}$$

   is a possible motion of an incompressible fluid.

   (i) Determine the equations of the relevant stream lines.

   (ii) Also test whether this motion is of the potential kind and if so find the velocity potential.

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2. Derive in the usual notations, the Euler equation of motion $\underline{F} - \dfrac{1}{\rho}\underline{\nabla}P = \dfrac{d\underline{q}}{dt}$ for a perfect fluid moving under a force $\underline{F}$ per unit mass and velocity $\underline{q}$.
Hence, by using suitable conditions deduce that

$$\underline{\nabla}\left(\frac{P}{\rho} + \frac{1}{2}\underline{q}^2 + \Omega\right) = \underline{q} \wedge curl\underline{q},$$

where Ω is the scalar potential such that $\underline{F} = -\underline{\nabla}\Omega$.
A quantity of liquid of density ρ occupies a length $2a$ of a straight tube of uniform small cross section. The liquid is under the action of a force kx per unit mass towards a fixed point O in the tube, where x is the distance from O.
Show that, when the nearest free surface is at a distance z from O, the pressure at a distance x exceed atmospheric pressure by

$$k\rho(x - z)\left(a - \frac{1}{2}x + \frac{1}{2}z\right).$$

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3. An incompressible inviscid fluid of uniform density ρ is in irrotational motion. Show in the usual notation, the kinetic energy T of the fluid enclosed by a surface s is given by

$$T = -\frac{1}{2}\rho \int_s \phi \frac{\partial \phi}{\partial n} ds,$$

where $\underline{n}$ is the outward unit normal vector to the fluid surface and ϕ is the velocity potential.
A solid sphere of mass M and radius a moves in a uniform liquid which is at rest at infinity. If the sphere is moving with velocity $\underline{u}$, choosing the origin and axis suitably, find the velocity potential at the point $p(r, \theta)$.
Also find the kinetic energy of the liquid.
(Assume that the velocity potential of a sphere in the usual notation as $(Ar + Br^{-2})cos\theta$.)

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4. a) Obtain the velocity potential of a three dimensional source.
b) Three dimensional sources of strength m_1 and m_2 are situated at points $(2a, 0, 0)$ and $(-2a, 0, 0)$ respectively.
(i) Find the resulting velocity potential $\phi(r, \theta)$ in spherical polar co-ordinates at a point $p = p(r, \theta)$ under the axial symmetry.
(ii) If $\dfrac{m_1}{m_2} = -\left(\dfrac{k-2a}{k+2a}\right)^2$; where k is a constant, show that there is no transport of fluid through $x^2 + y^2 = k^2$ on the plane $\theta = 0$.
(iii) Find the stagnation points on the plane $\theta = 0$.

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5. a) A source and sink of equal strength m are placed at the points $(2a, 0)$ and $(-2a, 0)$ respectively in a two dimensional flow.
(i) Write down the complex potential of the system.
(ii) Now above system is placed in a fluid moving with the constant velocity $-u\underline{i}$. Write down the new complex potential of the system.
(Here $\underline{i}$ is the unit vector in the direction of positive x-axis.)
b) Let the complex potential of a 2-D motion is
$W(z) = u\left(z + \dfrac{a^2}{z}\right) + iklog\left(\dfrac{z}{a}\right)$; where a, u and k are constants.
(i) Show that $q^2 = u^2\left(1 - \dfrac{a^2}{z^2}\right)^2 + \dfrac{k^2}{z^2}$. Hence find the velocity at infinity in $\overrightarrow{ox}$ direction.
(ii) Find the stream function of $|z| = a$.
(iii) Find the stagnation points.

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6. State the Milne-Thomson Circle theorem and its extension.

A source and sink of equal strength m are placed at the points $\left(\pm\frac{a}{3}, 0\right)$ within a fixed circular boundary $|z| = \dfrac{2a}{3}$. Find the complex potential of the system. Show that the stream lines are given by

$$48a^2y^2 - \lambda\left(9r^2 - 4a^2\right) = 3\left(r^2 - \frac{a^2}{9}\right)\left(9r^2 - 16a^2\right);\text{ where } \lambda \text{ is a constant.}$$
