University of Ruhuna
Bachelor of Science General Degree
Level III (Semester I) Examination – September 2017

SUBJECT: APPLIED MATHEMATICS/ INDUSTRIAL MATHEMATICS

COURSE UNIT: AMT 312β/ IMT 312– MATHEMATICAL MODELING III

INSTRUCTIONS:

- Show all work, simplify your answers and write out your work neatly for full credit.
- Answer **FOUR** questions only.
- Time Allowed: **TWO** hours.

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1. Define $L[y(t)]$, the Laplace transform of the function $y(t)$.

   (a) Let $y_1(t)$ and $y_2(t)$ be functions whose Laplace transform exist and let c_1 and c_2 be scalars. Show that $L[c_1y_1(t)+c_2y_2(t)] = c_1L[y_1(t)]+c_2L[y_2(t)]$.

   (b) Using the definition, find $L[e^{at}]$, where $a \in \Re$.

   Hence, find $L[c]$, where $c \in \Re$.

   (c) Find $L[2\sinh at + 5]$, $a \in \Re$.

   (d) Let $c \in \Re$ and let the *Heaviside step function* $u_c(t)$ be defined by

$$u_c(t) = \begin{cases} 0; & \text{when } t \le c, \\ 1; & \text{when } t > c. \end{cases}$$

   (i) Show that $L[u_c(t)] = \dfrac{e^{-sc}}{s}$.

   (ii) Let $f(t) = \begin{cases} 0, & \text{if} \quad t \le 1, \\ 1, & \text{if} \quad 1 < t \le 2, \\ 2, & \text{if} \quad 2 < t \le 3, \\ -1, & \text{if} \quad 3 < t \le 4, \\ 0, & \text{if} \quad t > 4. \end{cases}$

   It is given that $f(t) = u_1(t)+u_2(t)-3u_3(t)+u_4(t)$. Find $L[f(t)]$.

2. (a) (i) Using partial fractions, show that

$$\frac{1}{(s^2+1)(2s^2+3s+1)} = \frac{-3s-1}{s^2+1} + \frac{16}{2s+1} + \frac{-5}{s+1}.$$

(ii) Determine $L^{-1}\left[\dfrac{1}{(s^2+1)(2s^2+3s-1)}\right]$.

(b) Solve the following initial value problem:

$$2y'' + 3y' + y = 10\cos t; \quad y(0) = 1,\ y'(0) = -1.$$

3. (a) Let $y(t)$ be continuous and of exponential order and let $y'(t)$ be piecewise continuous on every finite interval.

(i) Show that $L[y'(t)] = sL[y(t)] - y(0)$.
Hence, find $L[y''(t)]$.

(ii) Let $f(t) = \sin bt$. Show that $L[-b^2 \sin bt] = s^2 L[\sin bt] - b$.
Hence, find $L[\sin bt]$.

(b) Consider two 100-gal tanks. Tank A is initially filled with water in which 25 lb of salt are dissolved. A 0.5 lb/gal salt mixture is poured into this tank at the constant rate of 4 gal/min. The well mixed solution from tank A is constantly being pumped to tank B at the rate of 6 gal/min, and the solution in tank B is constantly being pumped to tank A at the rate of 2 gal/min. The solution in tank B also exits the tank at the rate of 4 gal/min. Set up the system that will give the amount of salt in each tank at any given time.

4. (a) Draw
(i) a simple graph
(ii) a multigraph with one self-loop
each with ***six vertices*** and ***eight edges***.

(b) Draw the following graphs:
(i) the complete graph K_5;
(ii) the complete bipartite graph $K_{4,6}$;
(iii) the complement of the complete bipartite graph $K_{3,4}$.

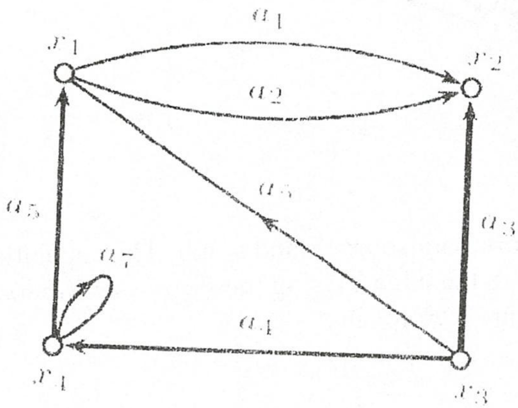
Cts...

(c) (i) State the *"Hand Shaking Lemma"*.

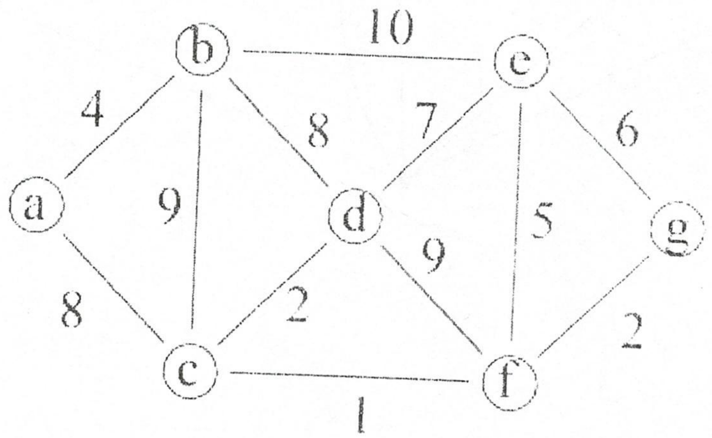
(ii) Hence, show that any simple graph has even number of vertices odd degree.

(d) Let G be a simple graph all of whose vertices have degree 3 and $|E| = 2|V| - 3$. Find the number of vertices of G and draw the graph of G.

5. (a) Write down the adjacency matrix and the incidence matrix of the following graph:



(b) Use the matrix version of *Prim's algorithm* to find a minimum spanning tree in the following weighted graph:



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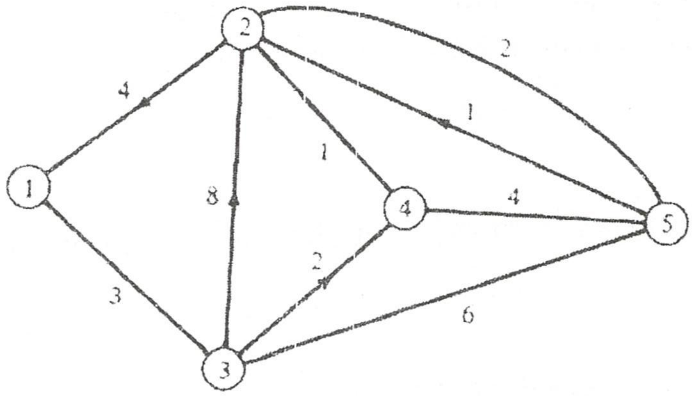
6. (a) The following network gives the routes and their distance in miles between five cities; 1, 2, 3, 4 and 5. Use the Floyd Warshall algorithm to find the shortest routes between any two cities.



(b) The graph below shows a network with source s and sink t. The capacities are indicated by numbers attached to the edges. Using the *Ford – Fulkerson algorithm*, find the maximal flow from source "s" to sink "t".