University of Ruhuna
Bachelor of Science General Degree
Level III (Semester I) Examination – September 2017

Subject: Mathematics
Course Unit: MAT313 β **Time: Two (02) hours**

Mathematical Statistics II

Answer 04 Questions only.

1.

(a) Let $Y_1, Y_2, \ldots, Y_n$ be a random sample of size n from a Poisson population and the probability mass function of Y is given by,

$$P(Y = y) = \frac{\lambda^y e^{-\lambda}}{y!}, \quad y = 0,1,2,\ldots$$

Estimate the parameter λ by using the method of moments.

Let Y be the number of paint defects found in a square meter section of a car body painted by a robot. The following data are obtained:

8	5	0	10
0	3	1	12
2	7	9	6

Estimate λ, assuming that Y has a Poisson distribution with parameter λ.

(b) Let X_1, X_2, X_3 and X_4 be a random sample of size 4 from the discrete distribution X such that,

$$P(X = x) = \begin{cases} \dfrac{\theta^{2x} e^{-\theta^2}}{x!} & \text{for } x = 0,1,2,\ldots \\ 0 & \text{otherwise} \end{cases}$$

where $\theta > 0$. Find the maximum likelihood estimator of θ.

2. State the factorization theorem for a sufficient statistic.

Let $Y_1, Y_2, \ldots, Y_n$ be a random sample of size n from a distribution with $P(Y_i = 1) = p$ and $P(Y_i = 0) = 1 - p$, where p is unknown. Show that $\sum_{i=1}^{n} Y_i$ is a sufficient statistic for p.

State the Rao-Blackwell theorem in the usual notation.

Hence show that $\hat{p} = \overline{Y}$ is the minimum variance unbiased estimator (MVUE) for p, where $\overline{Y} = \frac{1}{n}\sum_{i=1}^{n} Y_i$.

(vi) Sketch the distributions of $\overline{Y}$ under H_0 and H_1 in a diagram. Shade the regions relevant to α and β.

(vii) If the sample size is increased, the standard deviation of $\overline{Y}$ will decrease. What is the geometric effect of this on the two curves of the diagram in part (vi)?

(viii) If the sample size is increased but the critical point is not changed, what will be the effect on α and β?

5. Suppose that the distribution of the lifetime of TV tubes can be adequately modelled by an exponential distribution with mean θ, so that

$$f(x;\theta) = \begin{cases} \frac{1}{\theta} e^{-x/\theta} & \text{for } x \geq 0 \\ 0 & \text{otherwise.} \end{cases}$$

Under the usual production conditions, the mean lifetime is 2000 hours but if a fault occurs in the process, the mean lifetime will drop to 1000 hours. A random sample of 20 tube lifetimes is to be taken in order to test the hypotheses $H_0 : \theta = 2000$ versus $H_1 : \theta = 1000$.

Use the Neyman-Pearson lemma to find the most powerful test with the significance level α.