

**University of Ruhuna**  
**Bachelor of Science General Degree Level III**  
**(Semester II) Examination - January 2018**

**Subject: Mathematics**

**Course Unit: MAT327 $\beta$  (Introduction to Financial Mathematics)**

**Time :Two (02) Hours**

**Answer 04 Questions only.**  
**A calculator will be provided.**

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1. (a) Let  $a(t) = (t + 1)^3$ . Find  $\delta(t)$ .
- (b) Let  $\delta(t) = \frac{4}{(t+3)}$ . Find  $a(t)$ .
- (c) Ernie makes deposits of 100 at time 0 and  $X$  at time 3. The fund grows at a force of interest

$$\delta(t) = \frac{t^2}{100}, \quad t > 0.$$

The amount of interest earned from time 3 to time 6 is also  $X$ . Calculate  $X$ .

- (d) If  $\delta = 0.08$ , find  $i^{(6)} + d^{(4)}$ .

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2. (a) Let  $a_{\overline{n}|i}$  be the present value of the unit annuity immediate with  $n$  payments of 1 and interest rate  $i$ . Obtain the formula for  $a_{\overline{n}|i}$ . Write down a formula for  $\ddot{a}_{\overline{n}|i}$ , the present value of the unit annuity due.
- (b) A 10-year annuity due pays 50 quarterly for the first 5 years and 100 quarterly for the last 5 years. The annuity earns at a nominal rate of 6% convertible quarterly. What is the present value of this annuity?
- (c) An insurance company has an obligation to pay the medical costs for a claimant. Average annual claims costs today are \$5,000, and medical inflation is expected to be 7% per year. The claimant is expected to live an additional 20 years. Claim payments are made at yearly intervals, with the first claim payment to be made one year from today. Find the present value of the obligation if the annual interest rate is 5%.

3. (a) Let  $Da_{\overline{n}|i}$  be the present value of the unit decreasing immediate annuity whose  $n$  payments are  $n, n - 1, \dots, 1$ . Write down a formula for  $Da_{\overline{n}|i}$ .
- (b) Olga buys a 5-year increasing annuity for  $X$ . Olga will receive 2 at the end of the first month, 4 at the end of the second month, and for each month thereafter the payment increases by 2. The nominal interest rate is 9% convertible quarterly. Calculate  $X$ .
- (c) An annuity immediate pays 20 per year for 10 years, then decreases by 1 per year for 19 years. At an annual effective interest rate of 6%, the present value is equal to  $X$ . Calculate  $X$ .
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4. (a) Give two formulas to calculate the interest paid and the principal paid in the  $t^{\text{th}}$  payment of a loan repayment with level payments.
- (b) A 10-year loan of 2000 is to be repaid with payments at the end of each year. It can be repaid under the following two options.
- (i) Equal annual payments at an annual effective rate of 8.07%.
- (ii) Installment of 200 each year plus interest on the unpaid balance at an annual effective rate of  $i$ .

The sum of the payments under option (i) equals the sum of the payments under option (ii). Determine  $i$ .

- (c) A 40-year loan is paid with level annual payments at the end of each year. The principal paid in the 20<sup>th</sup> payment is 166.59 and the principal paid in the 25<sup>th</sup> payment is 244.78. Find the interest rate for this loan.
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5. (a) Consider a bond with face value  $F$  redeemable at par in  $n$  years. Let  $i$  and  $r$  be the annual interest rate and annual coupon rate for this bond respectively. Write down a formula for the price  $P$  of this bond in terms of premium and discount.
- (b) A three year \$1000 par bond has a coupon rate of 6% convertible semiannually. It is sold at a yield of 5% convertible semiannually.
- (i) Find the price of this bond.
- (ii) Is this premium or discount bond?
- (iii) Construct a complete amortization table for this bond.
- (c) A woman buys two 5-year 1000 par bonds. The first has 7.5% semiannual coupons and is priced to yield 8% convertible semiannually. The second has 6% semiannual coupons and is priced to yield 7% convertible semiannually. The coupon payments from two bonds are deposited in a fund that pays 6.8% convertible semiannually. What is her annual effective yield for this combined investment?
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