University of Ruhuna

Bachelor of Science Special Degree Level I & II (Semester II) Examination - January-2018

Subject: Statistics
Course Unit: MSP4263 (Design and Analysis of Experiments)

Time :Two (02) Hours

Answer ALL Questions.
A calculator will be provided.

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1. (I) Consider the randomized complete block design (RCBD) model

$$y_{ij} = \mu + \tau_i + \beta_j + \epsilon_{ij}, \quad i = 1, \ldots, a, \text{and} \quad j = 1, \ldots, b,$$

where y_{ij} is the observation under i^{th} treatment and j^{th} block, μ is the overall mean, τ_i is the i^{th} treatment effect, β_j is the j^{th} block effect and ϵ_{ij} is the random error with $\epsilon_{ij} \sim N(0, \sigma^2)$, σ^2 is a constant.

(a) Write down the sum of squares of the errors, L of y_{ij}.

(b) Obtain the least squares normal equations for the values of μ, and τ_i, and β_j that minimizes the L.

(c) Are these normal equations independent? Justify your answer.

(d) By solving, obtain the least squares solutions for μ and τ_i, and β_j. If you use any constraints, write them down.

(e) Obtain a formula for the ij^{th} residual.

(II) Write down the appropriate contrasts to the following hypotheses.

$H_{01} : \mu_1 = \mu_2.$
$H_{02} : \mu_1 + \mu_2 = \mu_3 + \mu_4$
$H_{03} : \mu_3 = \mu_4.$
Are these contrasts orthogonal?

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2. In the article "Calibration of an FTIR Spectrometer" (P. Pankratz, Statistical Case Studies for Industrial and Process Improvement, SIAM-ASA, 1997:19-38), a spectrometer was used to make five measurements of the carborn content (in ppb) of a certain silicon wafer on four consecutive days. The results are as follows:

Day 1 :	358	390	380	372	366
Day 2 :	373	376	413	367	368
Day 3 :	336	360	370	368	352
Day 4 :	368	359	351	349	343

(a) Write down a statistical model for this problem.

(b) Construct an ANOVA table. You may give a range for the $P-$value.

(c) Can you conclude that the calibration of the spectrometer differs among the four days at 5% level of significance?

(d) Estimate the overall mean and the treatment effects.

(e) Compute a 95 percent confidence interval estimate of the mean of Day 4.

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3. The article "Sprinkler Technologies, Soil Infiltration, and Runoff" (D. DeBoer and S. Chu, Journal of Irrigation and Drainage Engineering, 2001:234-239) presents a study of the runoff depth (in mm) for various sprinkler types. Each of the four sprinklers was tested on each of the four days, with two replications per day. It is of interest to determine whether runoff depth varies with sprinkler type; variation from one day to another is not of interest. The data are presented in the following table.

Sprinkler	Day 1	Day 2	Day 3	Day 4
A	8.3 5.5	7.8 4.5	10.7 9.8	10.6 6.6
B	6.5 9.5	3.7 3.6	7.7 10.6	3.6 6.7
C	1.8 1.2	0.5 0.3	1.7 1.9	2.2 2.1
D	0.7 0.8	0.1 0.5	0.1 0.5	0.3 0.5

Continued.

(a) Identify the blocking factor and the treatment factor.

(b) Construct an ANOVA table. You may give ranges fot the $P-$values.

(c) Can you conclude that there are differences in mean runoff depth between some pairs of sprinklers? Explain.

(d) Which pairs of sprinklers, if any, can you conclude, at the 5% level, to have differing mean runoff depths?

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4. (I) (a) In symbols, write down two 4×4 Latin square designs.

(b) An industrial engineer is investigating the effect of four assembly methods (A, B, C, D) on the assembly time for a color television component. Four operators are selected for the study. Furthermore, the engineer knows that each assembly method produces such fatigue that the time required for the last assembly may be greater than the time required for the first, regradless of the method. That is, a trend develops in the required assembly time. To account for this source of variability, the engineer uses the Latin square design shown below.

Order of Assembly	Operator			
	1	2	3	4
1	C=10	D=14	A=7	B=8
2	B=7	C=18	D=11	A=8
3	A=5	B=10	C=11	D=9
4	D=10	A=10	B=12	C=14

(i) Write down a statistical model for this problem.

(ii) Write down the analysis of variance table for this problem.

(iii) Analyze the data from this experiment with $\alpha = 0.05$ and draw appropriate conclusions.

(II) To assess the effect of piston ring type and oil type on piston ring wear, three types of piston ring and four types of oil were studied. Three replications of an experiment, in which the number of milligrams of material lost from the ring in four hours of running was measured, were carried out for each of the 12

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combinations of oil type and piston ring type. With oil type as the row effect and piston ring type as the column effect, the following sums of squares were observed: $SSA = 1.0926$ $SSB = 0.9340$ $SSAB = 0.2485$ $SS_{Res} = 1.7034$.

(i) Construct an ANOVA table, you may give ranges for the $P-$values.

(ii) Is the additive model plausible? Provide the value of test statistic and the $P-$value.
