UNIVERSITY OF RUHUNA

Faculty of Engineering

End-Semester 1 Examination in Engineering: October 2019

Module Number: ME1202 Module Name: Introduction to Mechanical Engineering

[Three Hours]

[Answer **all questions**, each question carries **12** marks]

Clearly state all assumptions that you may make.
To get full marks make sure that you have answered with correct SI units and standard notations.
Take **g = 9.8** N/kg.

Q1. i) A particle moving vertically down in the earth's atmosphere has an opposite acceleration given by the function of time ($a = f(t)$) and distance travels ($a = h(s)$) for two situations. If the displacement and the velocity of the particle are s_0 and v_0 respectively at time, t_0, obtain an expression for the velocity (v) of the particle after time, t for the
following two cases.

(a) When $a = f(t)$.

(b) When $a = h(s)$.

(4 Marks)

ii) The vehicle shown in Figure Q1 travels over a hill with a constant speed of 72 km/h on a path described by a function, $f(x) = 15 - 1.5 \times 10^{-3}x^2$ where $y = f(x)$ and both x and y are measured m. When $x = 50m$, determine

(a) the velocity and the acceleration vectors of the vehicle using $\underline{i}$ and $\underline{j}$ unit vectors along the x and y Cartesian axes,

(b) the magnitude and the direction of the velocity of the vehicle and

(c) the magnitude and the direction of the acceleration of the vehicle

(8 Marks)

Q2. i) Briefly describe what is linear momentum of a moving particle and state it in vector form.

(1 Mark)

ii) Mechanics based on the three laws of Newton considered as the Newtonian particle mechanics validated for a motion of a particle defined using an inertial reference frame. Briefly describe what is an inertial reference frame and give two examples for such systems.

(2 Marks)

iii) Based on above (i) and (ii), derive mathematically and explain the Newton's first law of motion and the Newton's second law of motion.

(3 Marks)

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iv) The system shown in Figure Q2 consists of a vehicle of mass M undergoing a rectilinear motion on a smooth surface due to a variable force, $\mathrm{F} = F(t)$. A pendulum consists of a lightweight non-elastic string (length l) and a particle (P) of mass m attached to its end, rotates about the center of gravity of the vehicle (C) freely.

(a) Define the axis system and draw free body diagrams for both the vehicle and the pendulum mass P showing all the forces acting on.

(b) Find the equations of motion of the system.

(6 Marks)

Q3. i) The position of the particle moving in a curvilinear path shown in Figure Q3(i) can be represented by its position vector $\underline{r}(t)$ at A with respect to the Cartesian coordinate system $x - y$. Radius of the curvature at A is R as measured from the center C as shown.

(a) Copy the diagram in Figure Q3(i) to your answer sheet and introduce normal and tangential $(n - t)$ coordinate system to describe the motion of the particle.

(b) Obtain velocity and acceleration vectors of the particle using unit vectors; $\underline{e}_n$ and $\underline{e}_t$ along the $(n - t)$ axes respectively.

(c) Sketch the directions of the normal, tangential and resultant acceleration of the particle when it is moving at finite and infinite radius of curvatures on a curvilinear path when the particle accelerating and decelerating.

(5 Marks)

ii). Figure Q3(ii) shows a non-smooth horizontal circular track of radius R fixed to $x - y$ plane and a circular collar of mass m that can slide counterclockwise in the track. The collar is given an initial velocity v_0 at its initial position P. The static and kinetic coefficients of friction between the collar and the track are μ_s and μ_k respectively.

(a) Draw the free body diagram of the collar showing all the forces acting on it when it is at the position Q after slides distance s along the track.

(b) Represent the friction force in vector form using a unit vector that can be defined using the collar velocity along the direction of motion.

(c) Using only the above given parameters, determine the distance travelled by the collar along the track when it comes to a rest.

(d) Show that the distance travelled in question (ii)(c) is inversely proportional to the coefficient of friction.

(7 Marks)

Q4. (i) (a) State the angular momentum about a fixed point (let's say O) for a system of particles in a 2D motion by taking position vector of i^{th} particle (mass m_i) is $\underline{r}_i$.

(b) Show that the rate of change of total linear momentum about a fixed point is equal to total moments about the same point. Hence obtain the condition for the conservation of angular momentum about a fixed point.

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(c) As shown in Figure Q4(i), A sphere (mass 6kg) and a block (mass 4kg, shown in section) are fixed to a lightweight arm which rotates in the vertical plane about a horizontal axis through O. The 2kg plug is released from rest at A falls into a recess (cut space to exactly fit or wedge the plug as shown in the section) in the block when the arm has reached the horizontal position. Arm angular speed just before the engagement of the block is ω_0.
Determine the angular velocity of the arm immediately after the plug has wedged itself in the block.

(7 Marks)

(ii) A pan of negligible mass is attached to two identical springs is shown in Figure Q4(ii). A 10kg box is dropped from a height of 50cm above the pan. Stiffness of each spring, $k = 250N/m$ and initially at the horizontal position (before weight touch to the pan) each spring has a tension of 50N.
Using the conservation of energy theory, show that the vertical displacement d that the pan reaches is given by the equation, $d^2 - \frac{1}{25}gd - 1.6\sqrt{d^2+1} + 351 = 0$.

(5 Marks)

Q5 i) Name and briefly explain main types of the planner rigid-body motion with suitable sketches.

(3 Marks)

ii) The T-shaped body rotates about a horizontal axis through point O. At the instant shown in Figure Q5(ii), its angular velocity (clockwise) and angular acceleration (counterclockwise) are $\omega\ rad/s$ and $\alpha\ rad/s^2$.

(a) Using the dimensions in the diagram and introducing a fixed Cartesian coordinate system appropriately, determine the velocity and the acceleration of point A and point B in vector forms (using $\underline{i}$ and $\underline{j}$ unit vectors).

(b) Express the same in terms of unit vectors $\underline{e_n}$ and $\underline{e_t}$ along n and t axes respectively.

(5 Marks)

iii) A piston-crank mechanism is shown in Figure Q5(iii) consists a crank of length R, a connecting rod of length L and smooth piston arrangement with all smooth joints. If the crank rotates at constant counterclockwise angular speed of $\omega\ rad/s$,

(a) Determine the limits of piston displacements measured from the fixed point, A (i.e. crank center).

(b) Determine the velocity and the acceleration of the piston for a given instant (when the crank makes an angle of θ).

(4 Marks)

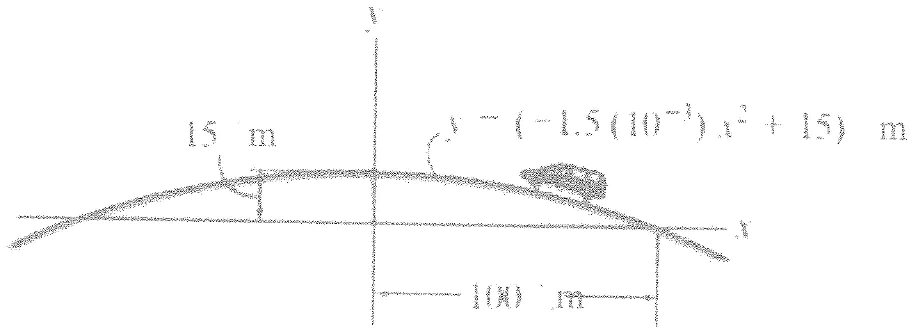

Figure Q1

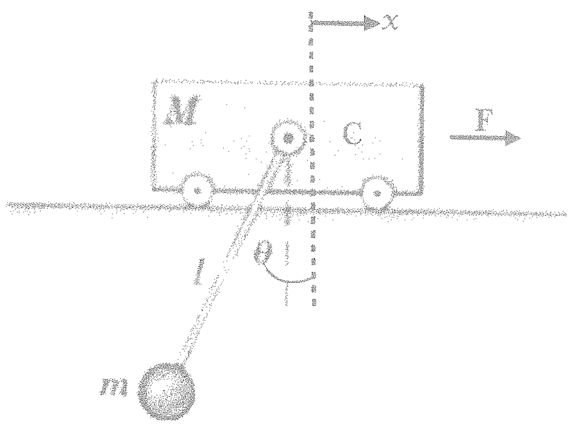

Figure Q2

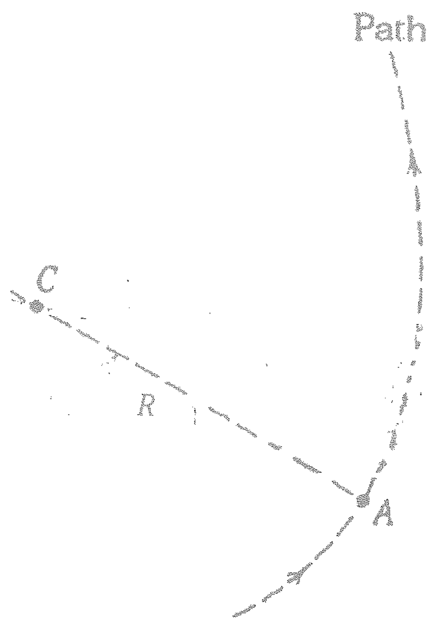

Figure Q3(i)

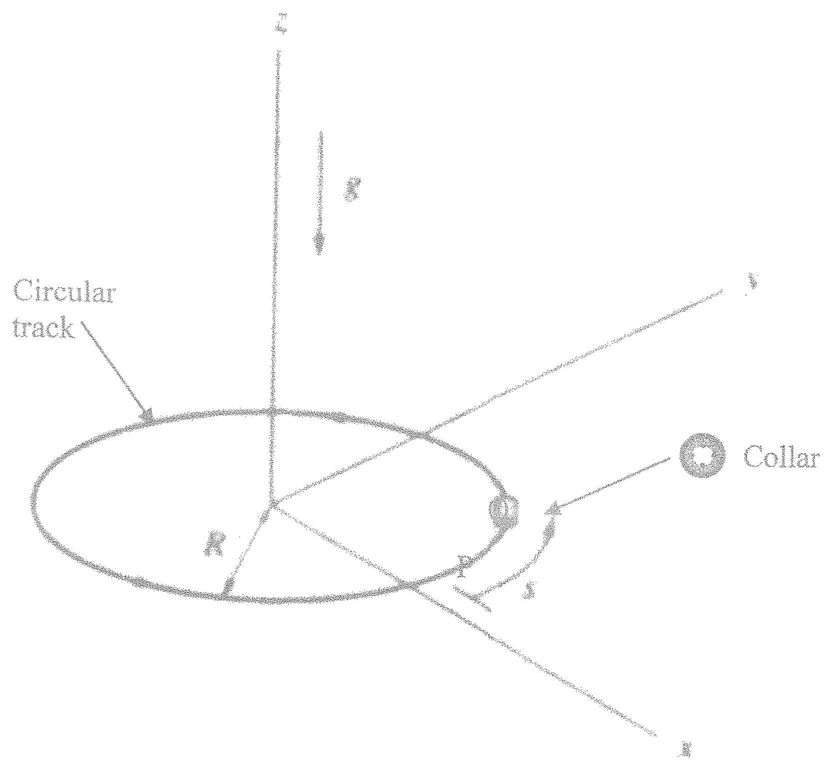

Figure Q3(ii)

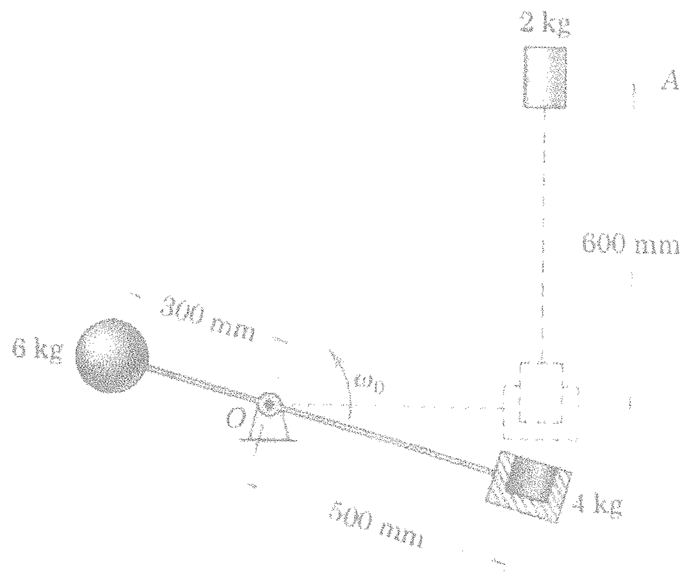

Figure Q4(i)

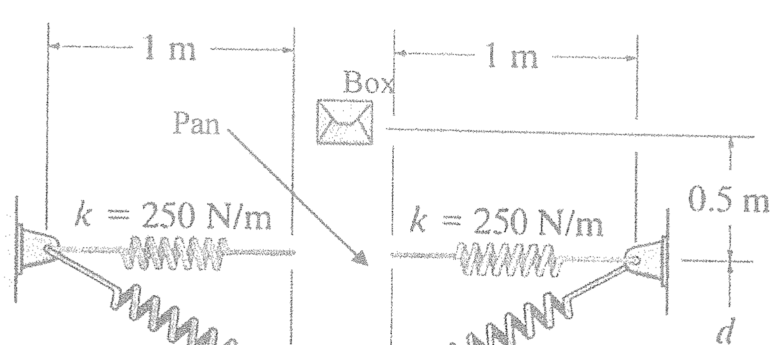

Figure Q4(ii)

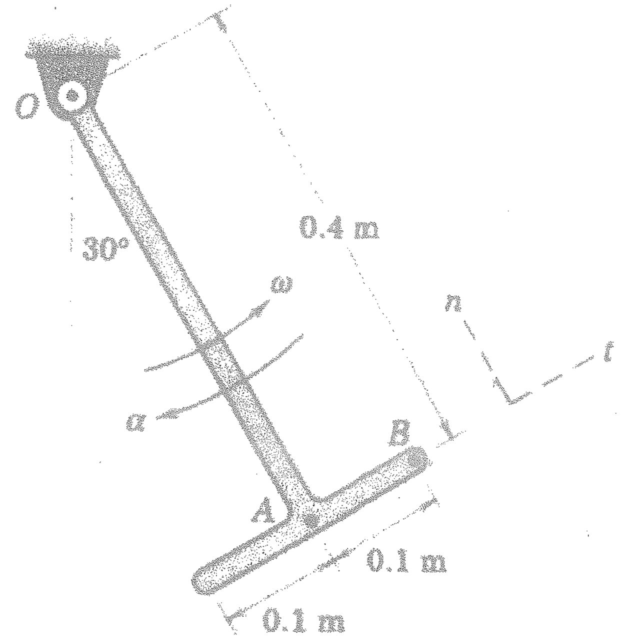

Figure Q5(ii)

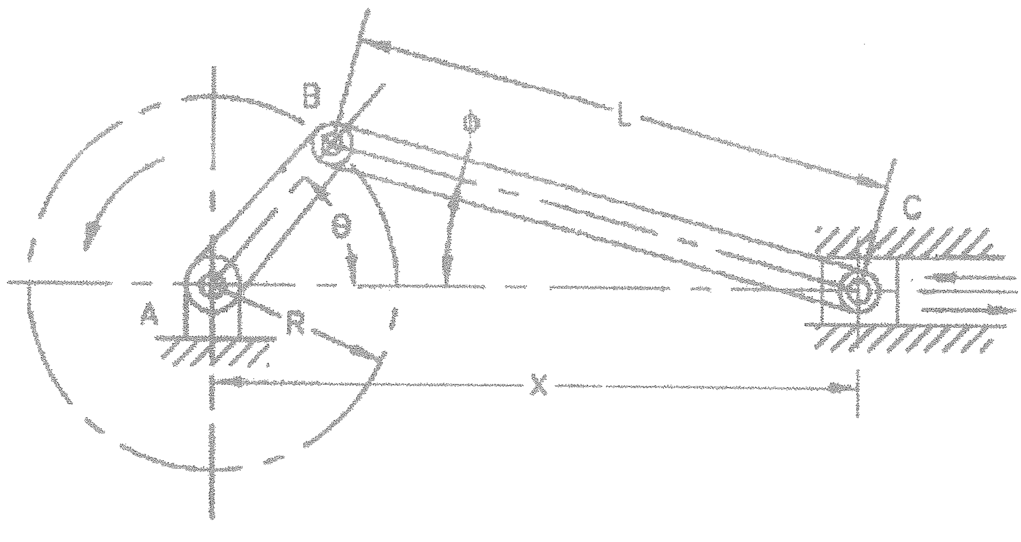

Figure Q5(iii)