UNIVERSITY OF RUHUNA

Faculty of Engineering

End-Semester 1 Examination in Engineering: October 2019

Module Number: IS1402 **Module Name: Mathematical Fundamentals for Engineers**

[Three hours]

[Answer **all questions**, each question carries twelve marks]

Q1. a) i.) Briefly explain what is meant by the modulus and the argument of a complex number $z = x + iy$, where x and y are real numbers.

ii.) Find the modulus and principal argument of

$$\frac{1+2i}{1-(1-i)^2}.$$

[2 Marks]

b) i.) State the De Moivre's Theorem, and prove the theorem only for the positive integers.

ii.) Use De Moivre's Theorem to simplify

$$\left(\cos\frac{5\pi}{4} + i\sin\frac{5\pi}{4}\right)^{10}$$

and express the result in standard form.

[3 Marks]

c) i.) Express $2^5 \sin^4\theta \cos^2\theta$ in terms of cosines of multiples of θ.

ii.) Use the complex root theorem to find all the solutions of the equation $x^3 + 64i = 0$ and represent the solutions graphically.

iii.) Determine the real and imaginary parts of $\log(x + iy)$.

[7 Marks]

Q2. a) Briefly explain the following types of matrices, and give an example for each type.

i.) Transpose of a matrix

ii.) Skew symmetric matrix

iii.) Hermitian matrix

iv.) Singular matrix

[2 Marks]

b) By using any square matrix (A) of order 3 or higher show that, $A(adj.A) = |A|I$, where $|A|$ is the determinant of the matrix A and I is a unit matrix.

[2 Marks]

c) Given $A = \begin{pmatrix} 1 & 1 & 1 \\ 2 & 4 & 1 \\ 2 & 3 & 1 \end{pmatrix}$, $B = \begin{pmatrix} 2 & 3 \\ 3 & 4 \end{pmatrix}$ find X such that $BXA = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$.

[3 Marks]

d) Find for what values of K, the system of equations

$$\begin{aligned} 2x - 3y + 6z - 5t &= 3 \\ y - 4z + t &= 1 \\ 4x - 5y + 8z - 9t &= K \end{aligned}$$

has

i.) no solution

ii.) infinite number of solutions

[5 Marks]

Q3. a) i.) Briefly explain what is meant by 'Unit vector' and 'Position vector'.

ii.) Determine the values of λ and μ using vectors, such that the points $(-1,3,2)$, $(-4,2,-2)$ and $(5, \lambda, \mu)$ are collinear.

[3 Marks]

b) A rigid body is rotating with angular velocity 2 radians/sec about an axis AB, where A and B are the points $(1,-2,1)$ and $(3,-4,2)$. Find the velocity of the body at the point $P(5,-1,-1)$.

[3 Marks]

c) i.) Find the divergence and curl of the vector field,

$$\underline{T} = (x^2 + yz)\mathbf{i} + (y^2 + zx)\mathbf{j} + (z^2 + xy)\mathbf{k}.$$

ii.) Find the directional derivative of $\underline{V}^2$, where $\underline{V} = xy^2\mathbf{i} + zy^2\mathbf{j} + xz^2\mathbf{k}$, at the point $(2,0,3)$ in the direction of the outward normal to the sphere $x^2 + y^2 + z^2 = 14$ at the point $(3,2,1)$.

[6 Marks]

Q4. a) Explain what is meant by

i.) An open interval

ii.) A closed interval

iii.) A semi open interval

[3 Marks]

b) i.) Explain what is meant by the function f is continuous at $a \in \mathbb{R}$.

ii.) Discuss continuity of the following function f on $\mathbb{R}$.

$$f(x) = \begin{cases} 3x + 1 & x \le -1 \\ x - 1 & -1 < x \le 1 \\ 2x - 1 & x \ge 1 \end{cases}$$

iii.) Write down an example for a real function f, where the limit of f exists every where on $\mathbb{R}$, but f is not continuous at 2.

[5 Marks]

c) Sketch the graph of $y = |x-2| + |x+1| - 2|x-1|$ for $x \in \mathbb{R}$.

[2 Marks]

d) Evaluate the following limits

i.) $\lim_{x \to 1} \dfrac{(x+1)^2 - x - 3}{x-1}$ ii.) $\lim_{x \to 0} \dfrac{x^2(\cos x - 1)}{\sin^2 x}$

[2 Marks]

Q5. a) i.) State the Rolle's Theorem

ii.) Show that, between any two roots of $f(x) = \cos x + \sin x$ there is a root of $g(x) = \cos x - \sin x$, where $x \in \mathbb{R}$.

[2 Marks]

b) i.) State and prove the Mean Value Theorem

ii.) Use Mean Value Theorem to show,

$$\frac{x}{x+1} < \ln(1+x) \quad \text{for all } x > 0$$

[5 Marks]

c) i.) Let $z = f(x,y)$, show that

$$dz = \frac{\partial z}{\partial x} dx + \frac{\partial z}{\partial y} dy .$$

ii.) State the Euler's theorem on homogeneous functions.

iii.) Use Euler's theorem to show that , if

$$u = \sin^{-1}\left(\frac{x^4 - 2x^2y^2 - y^4}{xy}\right)$$

then

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 2\tan u$$

[5 Marks]